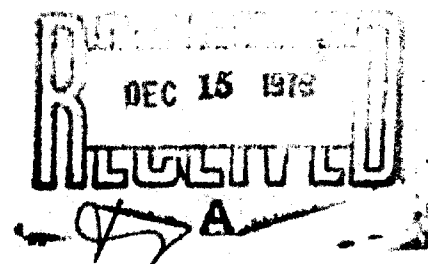


ADA 033523

12

Best Available Copy



DISTRIBUTION STATEMENT  
Approved for public release  
Distribution Unlimited

KETRON, INC.

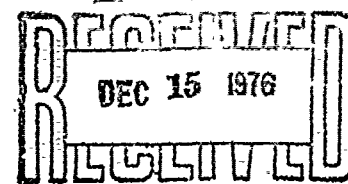
12

FIRE SUPPORT REQUIREMENTS  
METHODOLOGY STUDY PHASE II  
PROCEEDINGS OF THE FIRE SUPPORT  
METHODOLOGY WORKSHOP

KFR 57-75

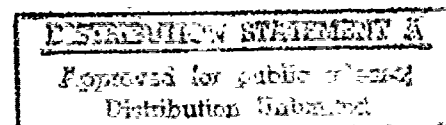
18 December 1975

Naval Postgraduate School  
Monterey, California  
4-7 August 1975



A

✓



This work relates to Department of the Navy Contract N00014-75-C-1144 issued by the Office of Naval Research under Contract Authority NR 277-097. However, the content does not necessarily reflect the position or the policy of the Department of the Navy or the Government, and no official endorsement should be inferred.

The United States Government has at least a royalty-free, non-exclusive and irrevocable license throughout the world for Government purposes to publish, translate, reproduce, deliver, perform, dispose of, and to authorize others so to do, all or any portion of this work.

UNCLASSIFIED

Security Classification

## DOCUMENT CONTROL DATA - R &amp; D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified

1. ORIGINATING ACTIVITY (Corporate author) Ketron, Inc. 1400 Wilson Boulevard Arlington, Virginia 22209		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	
		2b. GROUP N/A	
3. REPORT TITLE (6) Fire Support Requirements Methodology Study, Phase II Proceedings of the Fire Support Methodology WORKSHOP			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) (9) Final Report, 4-7 August 1975			
5. AUTHOR(S) (First name, middle initial, last name) (10) Ross S./Thackeray			
6. REPORT DATE (11) 18 Dec 1975		7a. TOTAL NO. OF PAGES 312	7b. NO. OF REFS
8. CONTRACT OR GRANT NO. (15) N00014-75-C-1144 NEW		9a. ORIGINATOR'S REPORT NUMBER(S) KFR-57-75	
9. PROJECT NO.		10. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) (14) N/A	
10. DISTRIBUTION STATEMENT Distribution of this document is unlimited.			
11. SUPPLEMENTARY NOTES N/A		12. SPONSORING MILITARY ACTIVITY Mathematical & Information Sciences Div Office of Naval Research (Code 434) Arlington, Virginia 22217	
13. ABSTRACT The U.S. Marine Corps initiated a multi-phased Fire Support Requirements Methodology Study (FSRMS) to improve the utility of their studies in support of decisions allocating resources to alternative mixes of future fire support weapon systems. Along with a review of existing models and techniques, a Fire Support Methodology Workshop was sponsored at the Naval Postgraduate School, Monterey, California, August 4-7, 1975. The Workshop brought together experts in methodology and fire support analysts to consider new techniques and the program of future development of fire support studies. This document records the proceedings of the Workshop.  Alternatives to simulation such as gaming and mathematical programming were discussed, as were the problems of modeling dynamic target designation processes, the functions of command and the effects of suppressive fires. The need for further methodological research was highlighted but a cautious approach to the immediate application of advanced methods was suggested. A hierarchy of simpler models in keeping with the validity of available inputs was recommended along with a reappraisal of what models can realistically be expected to contribute to decision making at senior staff levels.			

DD FORM 1473

1 NOV 65

(PAGE 1)

PLATE NO. 21856390 86

UNCLASSIFIED

S/N 0102-014-6600

Security Classification

**Security Classification**

14		Security Classification		KEY WORDS		LINK A		LINK B		LINK C	
						ROLE	WT	ROLE	WT	ROLE	WT



## TABLE OF CONTENTS

	<u>Page</u>
FOREWORD	11
Chapter I INTRODUCTION	1
Chapter II THE RESULTS OF PHASE I	4
Chapter III THE INVITED PAPERS	17
Chapter IV THE MINI-WORKSHOPS	23
Chapter V EDITOR's OVERVIEW	30

## APPENDICES

Appendix 1 Fire Support Methodology Workshop Attendees, Program & Participants

### INVITED PAPERS:

- Appendix 2 Framework for Effective Fire Support Analysis, by Bobick & Peters
- Appendix 3 Solution of a Large Scale Airforce Ordnance Planning Problem by Mathematical Programming, by Dantzig et al
- Appendix 4 The Role of Differential Models of Combat in Fire Support Analyses, by Cherry
- Appendix 5 The Contributions of Gaming and Game Theory in the Study of Fire Support Problems, by Shubik
- Appendix 6 Modeling and Markov Processes, by Sobel
- Appendix 7 Simulation as a Methodology for Fire Support Studies, by Fishman
- Appendix 8 Statistical Analysis of Simulations, by Iglehart

### REPORTS BY CHAIRMEN OF MINI-WORKSHOPS:

- Appendix 9 Target Generation and Designation, by Hinkle
- Appendix 10 Target Engagement, by Goettig
- Appendix 11 Fire Support System Mix Effectiveness Analysis, by Girard

### SUMMARIES:

- Appendix 12 Summary of Invited Papers, by Lieberman
- Appendix 13 Summary of Invited Papers, by Taylor

ACCESSION for	
NTIS	UNIT STATUS <input checked="" type="checkbox"/>
DDC	DATE STATUS <input type="checkbox"/>
UNCLASSIFIED	<input type="checkbox"/>
IDENTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODE	
P. 11	Avail. End of Status
A	

## FOREWORD

The Marine Corps has a continuing need for an analytical basis and a rationale for determining coordinated requirements for air, field artillery and naval surface fire support systems. Research and development studies are conducted to support programming, budgeting, force sizing and structuring decisions. The methodology utilized in the studies supporting the requirements must not only lead to sound decisions, but should also provide clear insights into the rationale for those decisions, both to the decision maker and those reviewing the decisions.

The Fire Support Requirement Methodology Study is an effort to determine the "best" methodology that the state-of-the-art can support. The "best" methodology is defined as being those techniques that will provide the most realistic results in a framework of Marine Corps constraints of time, personnel and money.

A description of the Marine fire support system, along with a review of applicable theories/techniques and fire support models, was provided as a background for the Workshop. Participants were both Office of Naval Research (ONR) contract theoreticians and representatives of agencies (military and civilian) active in fire support studies.

Because of the eminent workshop participants, this document is intended to reflect the state-of-the-art. Additionally, this report serves the purpose of conveying a better understanding of the Marine Corps fire support problem to the people who are most qualified to provide solutions to this complex problem.

## CHAPTER I

### INTRODUCTION

In common with other large-scale enterprises, the United States Marine Corps must divide limited funds among many alternative patterns of investment in such a way as to achieve the greatest feasible return. The necessary resource allocation decisions are complicated by the fact that the items of military hardware offered for purchase often have several non-unique capabilities; they are usually presented first as developmental concepts with initial operational capability (IOC) seven to ten years in the future and they generate their return on investment against an evolving opposition and in situations which are only vaguely predictable. Since the choice of fire support systems is a large and vital sub-set of their total allocation problem, it is not surprising that the Marine Corps has sought to use the techniques of systems analysis and management science to improve their selections of weaponry and associated gear. Unfortunately, this rather costly effort has been a disappointment. All too often, a study that had been well received, used and appreciated at lower organizational levels was subject to increasing criticism and even rejected when briefed to the senior staff. The main complaints centered on a lack of realism, especially in such areas as the portrayal of the target mix engaged by fire support weapons, the simulation of the suppressive effects of supporting fires, and the modeling of the flexibility available to a MAF commander in his conduct of a battle. To improve the contribution which formal studies could make in allocating resources to fire support systems, the Fire Support Requirements Methodology Study (FSRMS) was established.

The FSRMS is structured in three phases. Phase I is to provide a description of the existing Marine Corps Fire Support system, review the current state-of-the-art in the analysis of such systems, and outline a tentative methodology development plan. Phase II, the subject of this report, is an extension of the Phase I effort in the form of a one-week workshop that gathered the most competent professionals in the field to discuss possible innovations in methodology and to comment on the development plan of Phase I. Phase III, which runs concurrently, is to study the Marine Corps decision processes to determine the nature and extent of the objectives and constraints that these processes impose on the set of methodologies (Reference 1).

This document records the "Proceedings of the Fire Support Methodology Workshop" which was held at the Naval Postgraduate School at Monterey, California, 4-7 August 1975. The aim of the meeting was threefold. As stated in Reference 2, and verbally by the project officer, it was:

- a. "To gather the most qualified theoreticians (Gaming and simulation, large scale programming, applied statistics, etc.) and practitioners in the field of fire support". (Reference 2, page 1)
- b. To inform the academic and research communities of the Marine Corps' needs for improved methodologies in the study of fire support requirements.
- c. "To establish a dialogue, informed by the product of Phase I, that would lead to the approach that should be taken to achieve the desired methodologies". (Reference 2, page 1)

The program and the list of participants, their home organizations, and the mini-workshops to which they contributed, are given in Appendix 1.

The Fire Support Workshop was conducted in four stages. The first day was devoted to presentations on and discussions of the work of Phase I. The second day was devoted to invited papers on various

techniques and disciplines which might improve current methodologies. On the third day, the participants broke up into seminar groups or mini-workshops on the subjects of "Target Generation and Detection", "Target Engagement", and "Fire Support System Mix Effectiveness Analysis". The final day was devoted to further discussions and summarization. The pattern of the program is reflected in the organization of this paper. Following this introduction are chapters on the results of Phase I, summaries of the invited papers, reports on the deliberations of the mini-workshops, and finally, the observations and conclusions of the editor. The presented papers appear, in full, in the appendices.

Since the results of Phase I were the starting point for much of the discussion in the Phase II Workshop, the highlights are summarized in Chapter II. This is only done as a convenience, however, and does not pretend to be a substitute for a study of the three reports which record the work of the first phase.

## CHAPTER II

### THE RESULTS OF PHASE I

Phase I of the FSRMS, which was reviewed by the speakers on the opening day of the Workshop, was done under contract by the Potomac General Research Group (PGRG). It consists of three major tasks, which were reported in references 1-3 and summarized in reference 4. These tasks were given descriptive titles as follows:

Task 1 - Verbal and Mathematical Description of the Marine Corps Fire Support System.

Task 2 - Review of Theories and Techniques Applicable to Marine Fire Support.

Task 3 - Survey of Programmed and Operational Models for Evaluating the Fire Support System.

#### Task 1

The descriptions of Task 1 provide the friendly organizational and operational settings within which candidate fire support weapon systems must be evaluated. They are therefore an elastic framework for fire support requirements studies in that novel weapons systems may require changes in the settings to achieve their greatest effectiveness. The descriptions also make clear the large numbers of target sensors, specific weapons and munitions whose varied characteristics must be adequately modeled by any useful analytic methodology. But perhaps the most important outcome is the graphic demonstration of the complex and tight interaction which exists between all elements of the fire support system, the command and control system and the actual progress of the engaged forces to be supported.

To make the fire support system more amenable to modeling and analysis, it is broken down conceptually into three fundamental subsystems:

1. Target generation sub-system. This sub-system is composed of all sensors or information sources used to detect the presence of targets and the processes for turning detections into lists of potentially useful targets. In general, the processing of detections tends to be in pre-planning, rear-to-front, during the preparatory phase prior to an amphibious assault. Targets are isolated by remote means or intelligence. During preparation for a combat operation by forces already ashore, preplanning still predominates, but the process is front-to-rear. Once combat is joined, preplanning gives way to improvised responses and the bulk of the increasing number of target detections is by ground observers. Fire Support Coordination Centers (FSCC) exist at each level of command to carry out the processing.

2. Target designation sub-system. This is the sub-system which acts on the flow of potential targets from the generation sub-system to determine which ones should be attacked by supporting arms, and with what priority. The process is supported by an elaborate communications network. In less urgent situations, it is a deliberate process with each echelon making its input. When a fast response is imperative, the engaged unit requesting support can talk directly to the supporting unit with intermediate FSCCs monitoring in a silence-means-consent mode.

3. Target engagement sub-system. Targets to be attacked have restrictive characteristics, such as distance, proximity to friendly forces, short life span, and so on. This sub-system surveys the available types of supporting fires (air-borne, artillery and naval) and selects the one most nearly meeting requirements. Equipment and munitions within the type are then matched to target vulnerability and defenses and the mission is ordered. The loop is closed by the assessment of the degree of mission accomplishment.

Much of the discussion of techniques and modeling which occurs in later tasks of Phase I and in Phase II relates directly to the fidelity with

which various methodologies can reproduce the performance of these subsystems. This is because the "target list" against which the simulated supporting fires are directed is the driving element in any study of fire support requirements.

## Task 2

### Framework for Analysis

"Fire Support Requirement Studies" is a title for a broad category of analytic effort. It can include studies to help in choosing the best weapon against a particular target. It can, at the other extreme, mean attempting to determine the most cost effective evolving total force structure for the foreseeable future. To keep this breadth of meaning from being a source of confusion, as it has often been, Task 2 was begun by defining six levels of analysis. The frequency with which these levels were referred to by the participants in Phase II attests to the utility of the definitions. They are as follows:

Level 1 - Engineering Performance Characteristics. This is a descriptive analysis defining the engineering features of an individual weapon or weapon system. For quantitative studies at higher levels, it provides basic performance inputs.

Level 2 - System- Subsystem Performance. This level is concerned with system performance measures, not just physical parameters. It should include the human element as in such factors as suppressive effect of supporting fires. Typical measures of effectiveness (MOEs) tend to be ad hoc and arbitrary and might include coverage area, response time or weighted kill scores against various target mixes.

Level 3 - Combat Effectiveness. Here, the attempt would be made to evaluate effectiveness of fire support in terms of its contribution to overall mission success in combined arms combat. Though some models of combined arms operations do exist, they are not suitable for fire support analysis and have not been used. Level 3 analysis is highly



scenario-dependent and is usually skipped in proceeding to higher levels.

Level 4 - Sensitivity Analysis. Analysis of sensitivity of results to variations of input parameter values is required at every level. In this discussion, stress is placed on sensitivity of results to the variability of enemy threats in the context of different scenarios. This could be done by comparing results of many Level 3 analyses but it is very expensive and the problem of the relative value to be attached to each scenario remains.

Level 5 - Force Mix Analysis (a snapshot in time). This level derives force mixes which perform well in a variety of settings against many threats at some specified future time period. It does not yield "optimal" mixes for any one threat or scenario and it ignores the course of events both before and after the specified time period.

Level 6 - Time-Phase Force Mix Analysis for Procurement Decisions. This level seeks to develop a time-phased procurement plan which takes account of feasible acquisition schedules and assures that the best overall mix of weapons is available at all times out to the ultimate planning horizon.

With regard to these six levels of analysis, several observations were made. Constraints on time and resources and a lack of completely adequate methodologies have generally restricted fire support analyses to Levels 1, 2 and 5. The most productive analysis is at Level 2. Full support of many real-world procurement decisions would require studies which reach Level 6.

Because studies at any level must base their assessments on value judgments drawn from a higher level, there is no possibility, even in theory, of formal studies or computer techniques alone making valid determinations of truly optimal fire support weapons mixes. Judgment is necessary in evaluating and applying the results of studies done at

all levels. In particular, the results at Levels 5 and 6 have meaning only in the light of judgments regarding the relative national importance of differing threats and scenarios.

Methods and Techniques (Strengths and Weaknesses):

Noting that Level 1 problems are manifest in higher levels, the discussion of techniques moved directly to Level 2. Here, the first major problem is to find ways of developing a realistic target list against which to apply the candidate weapon systems. In the real world, the target list is extremely dynamic. One side develops sensors, tactics and procedures for target detection. The other side develops countermeasures to frustrate them. The accumulating effects of friendly fires alter the target list - but so do enemy reinforcements and efforts to harden facilities. Not the least source of the dynamic nature of targeting is the fact that the priority for attack of each item on the target list is a strong function of the commander's battle plan and scheme of maneuver, as modified by the unfolding two-sided contest on the battlefield. If Level 3 analysis is to be attempted, it is imperative that the target list, which drives all that follows, faithfully reflect all of the salient features of the real world-targeting process.

Since the target list does drive all subsequent analysis, it is surprising that, in existing models, the target generation and designation subsystems are combined and both are isolated from the target engagement subsystem. The target list is then developed without dynamic feedback of the effectiveness of supporting fires. Coupled with a lack of adequate sensitivity analyses, this failure has led to rejection of results of prior studies. Possible improvements in target list development are offered by three techniques:

a. Extrapolation of History. This is the application of military judgment to combat experience. However, available historical data

are of doubtful validity for assessment of complex interactions and lose utility for studies set in future time frames.

b. War Games. These permit free exploration of the possibilities but are costly, do not reproduce results and leave uncontrolled such variables as the skill of the players, their motivation and the effect of outside influences such as controls and game structure.

c. Kinematic Analysis. This is a high resolution map exercise using actual topography and the real or projected physical characteristics of the weaponry. It permits two-sided, dynamic generation of targets but is cumbersome, time consuming, scenario and tactically dependent, and difficult to amalgamate with a computer simulation.

Given a target list, the modeling of the actual engagement of targets is in a somewhat better state. The modeling of weapons selection and the estimation of damage produced is generally adequately done by approximate formulae, but the input values remain rather suspect due to a lack of detailed data on these functions under field conditions. The really glaring deficiency is the lack of any adequate representation of the suppressive and neutralizing effects of supporting fires. It is generally agreed that these effects are the most militarily important, yet existing models deal only in kills or damage estimates. Since modeling these effects is conceptually straightforward, the continuing lack is seen to be the result of a basic lack of understanding of these effects as they exist on the battlefield.

Turning to Level 3, the Task 2 discussions note the difficulties of modeling a combined arms battle in itself and especially the difficult problem of representing the commander's role. This is the reason that in most prior fire support studies the problem has been finessed. Level 2 analyses are done to yield comparisons among weapon systems applied against a set of fire-support targets. These performance estimates are

then assumed to be the same as the system effectiveness measured as a contribution to a combined arms battle. In this way, Level 2 results are used as a surrogate for Level 3 analysis.

The need for Level 4 analyses is universally agreed to, but, in practice, they are almost never adequately done. Appropriate design of a hierarchy of models may offer a way to make them less costly.

Analysis at Level 5 is a "snapshot" substitute for a time-phased Level 6 analysis and in the past has been dependent on a Level 2 analysis used as a pseudo-Level 3 analysis. Even if full Level 3 analysis were available, the snapshot choice of a best mix depends on having comparisons among all possible candidate mixes, which generally are too costly to generate. This last problem has been eased by using regression analysis on the MOE sample points of candidate mixes to establish estimated smooth curves, and mathematical programming to find a constrained solution that reached at least specified minimal performance levels.

#### New Theories and Techniques:

The report of Task 2 (reference 2) discusses four techniques which are either new in their own right or which would be innovations in the treatment of fire support problems. These are Dynamic Planning, Optimization by Mathematical Programming, Value-Driven Combat Simulations, and Algorithmic Modeling.

Dynamic Planning. This concept was presented on the opening day of the Workshop by Dr. Ronald New as a way of moving from the "snapshot-in-time" results of Level 5 analysis to real support of the turbulent procurement process in Level 6. The mechanisms of dynamic planning were discussed in contrast to current study procedures.

In prior fire support requirements studies, it has been typical that a time period some years in the future was picked and a good average weapons mix determined for some subsequent life-cycle. This frozen snapshot tends to obscure the effects of increasing obsolescence of existing assets,

detail but is costly and non-reproducible. In his paper on Value-Driven Simulations, Dr. George Pugh describes an approach to machine modeling of the human decision process that mitigates the objections to previous techniques.

A canonical decision process, applicable to every-day human decision making, is described as follows:

1. Data input - Information is collected on the current problem.
2. Updating the model - The informal mental model of the problem in hand is revised to accommodate the new information.
3. Search for alternatives - Feasible and promising alternative courses of action are identified.
4. Simulation - The updated mental model is used to simulate the outcome of each course of action.
5. Evaluation - The outcomes are evaluated in terms of a value structure.
6. Decision - When an acceptable outcome is identified, the search is stopped. Until then the process is recycled to step 3.

In real life, however, it is frequently impossible to see one's way through to the ultimate end of an activity, be it a chess game or a combined arms battle. Hence, people actually use mental models of only part of the total activity and apply surrogate value systems, such as relative casualties, to stand in for final values, such as success or failure in a military campaign. By identifying the partial models and surrogate value systems actually used by experienced commanders (or better ones if possible) a basis is laid for greatly improved simulation of the commander's functions. However, to support this improvement, it is necessary to play in greater detail the acquisition and flow of intelligence.

Actual items of intelligence are the ingredients to the formation of a commander's view of his situation vis-à-vis the enemy. However, some of the required items are missing, incomplete, delayed or wrong. Their value decays with time. The result is that the commander's picture of his

changes in the threat, fluctuations in the budget or new R&D results. If taken literally, such a study would also imply a serious discontinuity in force structure at the beginning of the new weapon life-cycle.

New proposes that a continuous adjustment of force structure should be approximated by breaking the planning period of say, 10 to 15 years into natural segments, such as budget cycles, to achieve the resolution necessary for the problem in hand. At the beginning of each segment, careful attention would be directed to the current value of inherited assets. The decision criteria would include the value of the chosen mix as an input to the next segment. The process, of course, is repeated out to the planning horizon. In the application of this concept, it was stressed that success is critically dependent on adequate sensitivity testing of the impact on the results of both uncertain input values and any possibly controversial assumptions. Since this sort of planning is already inherent in the resource allocation procedures in the services, if only to control cash flow in the out years, this proposal would serve to bring studies into closer and more useful conformity to the decision-making processes they are intended to support.

Optimization by Mathematical Programming. On very few occasions have the powerful methods of mathematical programming been employed to derive constrained optimum mixes of fire support weapons. In reference 2 it is pointed out that more extensive application of the techniques may, by concentrating on force mixes, by-pass currently vexing problems such as explicitly modeling synergistic effects between weapon systems. The discussion of programming was considerably expanded in Phase II and will be amplified in later chapters.

Value-Driven Simulations. As pointed out earlier, Level 3 analysis places a premium on adequately modeling commanders' decision processes. So far, the processes have been represented in machine simulations by over-simple rules-of-thumb. In war games, decision-making is played in

situation is "fuzzy". To model the process of integrating intelligence items into a "fuzzy" picture, Pugh proposes to apply the techniques of Bayesian inference to stylized arrays of intelligence data. In this way, for the first time, he achieves explicit treatment of the effects of the fog-of-war within the canonical decision process.

#### Algorithmic Modeling

This technique was described in somewhat general terms by Mr. Timothy Horrigan. It is fundamentally an effort to return to computer machine language to devise computer representations of the elements of combat which are 100 to 1000 times more computationally efficient than similar representations in the usual readable, flexible simulation languages, such as Simscript. Program modules capable of achieving these economies are now available.

The benefits of such large improvements in efficiency are in the increased resolution of the combat simulations. Relevant events such as hits in small arms fire or combatant-to-combatant detections can be individually resolved, so that the inputs can be at the level of systems characteristics which are inherently more readily available and more reliable than aggregated performance measures. There are also obvious advantages in being able to trade, on more favorable terms, between detail of simulation, scope of study and computer running time. Such improved computational efficiencies could also be the key to obtaining adequate sensitivity analyses.

Improvements of technique by factors of 100 or 1000 are not achieved without cost. The report notes that to use the algorithmic models fire support studies must be recast and the familiar conceptual organization for land combat, Monte-Carlo simulations abandoned. There is also the cost and uncertainty attendant on taking the program modules from theory to practice. However, it is noted that what is offered is a potential breakthrough as opposed to an alternative of ever-larger, longer running models which are more expensive and no more credible.

### Task 3

The Task 3 survey of existing models applicable to fire support studies did not materially influence the discussions of the Phase II Workshop and the survey will not be reviewed in detail here. However, the conclusions of that task effort are summarized in reference 3, as follows:

"The existing models that are relevant to answering questions concerning the composition and effectiveness of the Fire Support System (FSS) are of two basic types. Those that focus on the processes of fire support, excluding the interacting combat systems (FSS models) and those that examine the performance of the force as a whole but not the FSS in detail. The former group includes the MAF model, the Legal Mix IV model, the Class V model and the DAFS/CAS model. The force performance models are BALFRAM, IDAGAM, VECTOR and LULETIAN.

We have tentatively concluded that the force performance models have very little utility in answering Marine Corps fire support questions. They abstract combat to such a degree that effectiveness differentials among closely competing fire support candidates would be lost in the "noise" of the output. Moreover, many of the inputs to these models are precisely the answers we are seeking; such as, the firepower effectiveness of a certain artillery battery. However, these models may have utility in selecting appropriate scenarios that should be used. It appears that for some time to come it will not be practical to rerun a FSS model against a sufficient number of scenarios to convince the user that a robust solution has been found. Under these circumstances, the force performance models may assist in sorting out the few scenarios that should be used. The Marine Corps is exploring this possibility at this time."



### Tentative Methodology Development Outline

As a strawman for consideration and discussion during the Phase III Workshop, the PGRG prepared a tentative program for near term and long range improvements in the methodology of Marine Corps' fire support studies. This proposal may be summarized as follows:

Near Term. Improvements to be available during FY 77.

1. The Target List. Major effort is proposed to develop a target list that: "(1) incorporates a realistic target detection routine; (2) carefully derives the target list from a detailed microscopic map analysis (kinematic analysis) of the deployed enemy; and (3) does so with the thoroughness that will permit altering the deployment of the defender and still be able to derive a credible new target list to correspond to the new tactical situation."
2. Kinematic Analysis. One to three scenarios and enemy force composition should be agreed on with Marine Corps professionals. Then the target generation and designation sub-system of an existing fire support model should be subjected to kinematic analysis down to squad or platoon level over a combat period of several days or weeks. This is estimated to require 3-4 man-years over a 9 month period. The scenarios of maneuver for each scenario would be developed in open games and related in detail to the target lists in order that, at a later time, the lists could be revised as appropriate for a different version of the fire support weapon system.
3. Suppressive Effects. Attempt should be made to improve the representation of suppressive effects of supporting fires. Perhaps the historical record would permit comparison of times to conduct a given operation when under such fires and when free of them. Also, field experimentation should be reviewed. An estimated 6-12 total man-months would be required.

4. Damage Equations. Research is recommended to improve the knowledge of fragmentation patterns, energies, wound ballistics, and the influence of terrain and posture on the effectiveness of each fire support munition. This is needed to validate the generally employed defeat-criteria concept.

Long Term Efforts. Improvements for use Post-77.

1. Basic Problem. To get to a valid Level 3 analysis, the full, combined arms battle must be modeled. The only reasonably convincing techniques for this are war games and computer simulations.

2. Simulations. Simulations are still hampered by a lack of suitable aggregations of the operations of small units up to the company on which to base the rules of a manageable Marine Amphibious Force (MAF) level model. The feasibility of very highly detailed simulations depends on the development of better representation of decision making and more efficient computational procedures. To achieve these goals, work on the development and application of value-driven combat simulations and algorithmic models is recommended.

### CHAPTER III

#### THE INVITED PAPERS

To broaden both the appraisals of Phase I and the search for new approaches, seven eminent individuals or groups in the forefront of methodological advances in Operations Research were invited to prepare special presentations. These were to be on the general Marine Corps problem as seen from their perspective, as well as on the innovative application of new techniques. Two other, equally well qualified, individuals were asked to summarize and comment on the invited papers. The full review papers by Professor Gerald J. Lieberman, Chairman of the Department of Operations Research, Stanford University and Professor James Taylor, Department of Operations Research and Administrative Science, Naval Postgraduate School, appear as Appendices 12 and 13 respectively.

The invited papers fall in three general categories: a review of what must be included in an adequate fire support analysis, examples of completed models having potential application to fire support requirement problems -- either directly or as methodological prototypes -- and general discussions of the possible utility of specific techniques.

#### Framework for Analysis

The first category is represented by a paper by J. C. Bobick and L. S. Peters of the Naval Warfare Research Center of SRI, entitled "Framework for Effective Fire Support Analysis" (Appendix 2).

The proposed framework stresses that several levels of decision making are covered in discussions of "the fire support problem". However, the analyses which support each level can be broken down into three major elements: the decision interface, the quantitative analysis, and the data reservoir. This structure is illustrated in Figure 1. From the discussion of each of the elements, the reviewers picked four points for citation:

- a. No further model building should be undertaken by the Marine Corps until a satisfactory fire support analysis structure is developed.

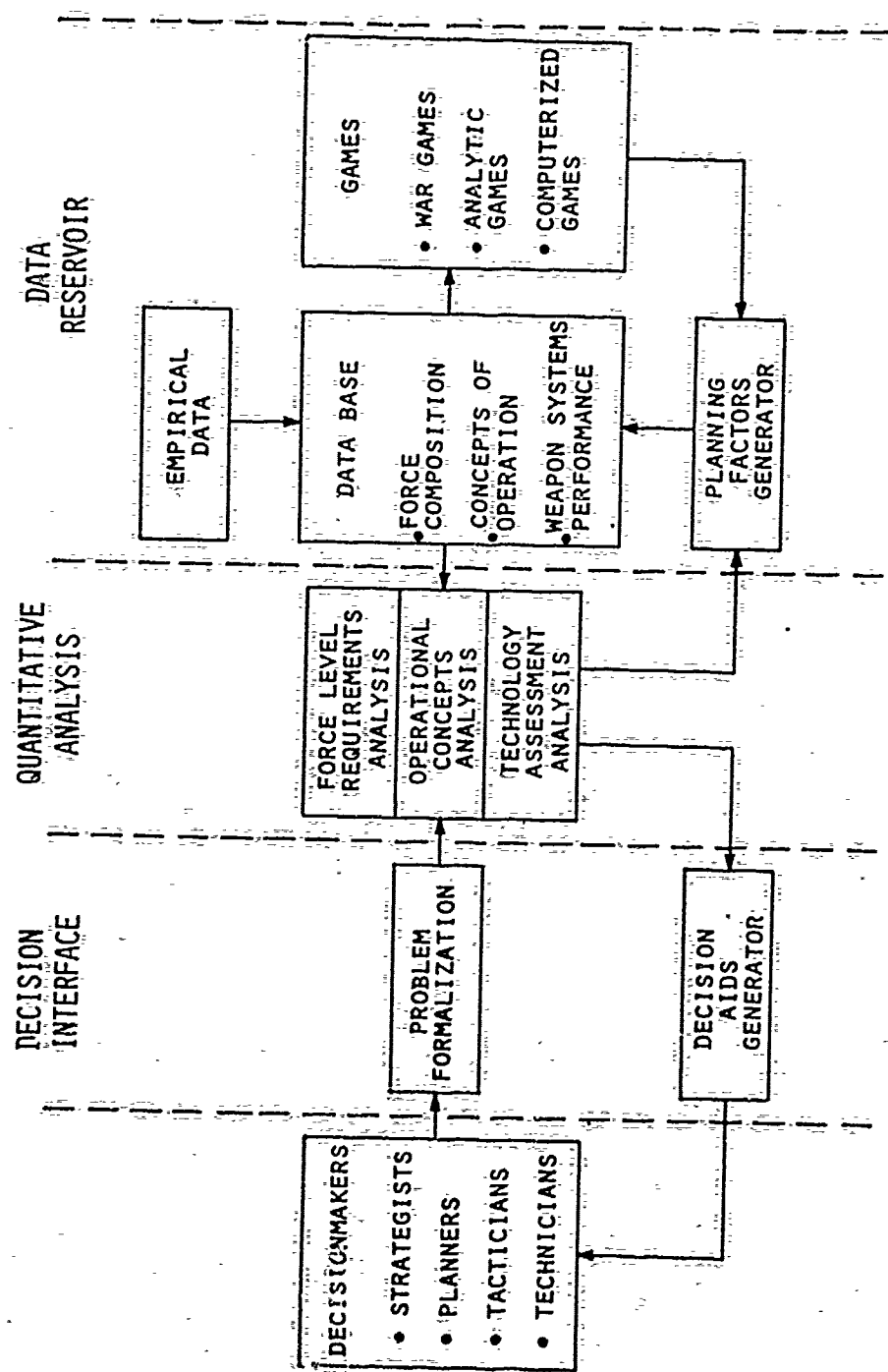


FIGURE 1 FRAMEWORK FOR EFFECTIVE FIRE SUPPORT ANALYSIS

b. Work is required to better define the decisions to be supported by analyses and to choose the appropriate MOEs for each level of decision making.

c. No single model can fulfill all of the Marine Corps requirements and the development of a hierarchical set of models is recommended.

d. A data reservoir should be established which provides for the accumulation of information necessary for the preparation (and standardization? Ed.) of inputs to fire support studies and the pooling of the results of completed analyses.

#### Completed Studies as Prototypes

The papers which dealt with complete studies were the "Solution of a Large Scale Airforce Ordnance Planning Problem by Mathematical Programming" (Appendix 3) given by George Dantzig and "The Role of Differential Models of Combat in Fire Support Analyses" (Appendix 4) which was presented by Peter Cherry.

The purpose of Dantzig's paper was to show, by example, that there is an alternative to simulation for the treatment of large-scale problems involving the optimization of a mix of resources. The example was a program written for the USAF to guide the selection of stocks of War Reserve Materiel (WRM). It optimizes, through maximizing targets destroyed after each of several stages of combat (typically 10, 30, 50, 100 days), the ordnance which was stockpiled before hostilities. Variables include aircraft types, missions, targets, delivery conditions, weather and time period. There is provision for 35 constraints on effectiveness such as sortie rates, attrition rates or budgets. The resulting convex, non-linear program is treated by piecewise linear approximations with software capable of handling 400 equations with over 500,000 variables. A problem of about 250 rows, 100,000 variables per time period and 10 periods takes about 75 minutes CPU time on an IBM 360/75 computer.

The process of optimization by stages was questioned for application in fire support requirements studies and was defended by Dantzig. He noted

that the sub-optimal solution could be compared with results of other simulation schemes. "If such stage-by-stage procedures are not good, he suggests the alternative scheme of simplifying the model until it can be solved by a dynamic program". However, the WRM model is static with fixed target lists, attritions per sortie and sortie effectiveness values. It was not immediately clear that the approach would succeed in overcoming the deficiencies of present fire support methodologies which demand an adaptive opponent and a dynamic target list.

Cherry's presentation covered the Vector Corporation's work on the theater level combat models Vector 0; Vector 1 and its derivative, the DIVOPS model; and the prospective Vector 2, with its increase in detail. Of these, only the last was judged to have sufficiently high resolution to be of use in fire support studies. None of these models contain optimization procedures for a weapon mix or tactics. Again, while the quality of the work for its original purpose was appreciated, the utility of these models for fire support studies has not been demonstrated, in spite of the initial hopes held by one of the reviewers for the applicability of DIVOPS.

#### Specific Techniques

Turning to the papers on the specific techniques of analysis and their application to fire support problems, Martin Shubik discussed the contributions to be expected from Gaming and Game Theory (Appendix 5). Beginning with a review of the distinguishing features of game theory and gaming, the benefits and pitfalls of each technique were outlined. He closed with some comments on the place of modeling within the decision making process.

Specifically, Shubik noted that game theory solutions were available only for oversimplified situations. Nonetheless, such games are a good way to begin the modeling of real world problems as complex as fire support operations, since they can clarify the importance of variables to be represented in the final simulation. Gaming, on the other hand,

draws both the sponsors and users into the full problem, including the human behavioral aspects, whether rational or not. On the general subject of modeling, the point was made that the unsophisticated often confuse "relevance" and "realism". This struck a responsive chord and was widely referred to by other participants.<sup>1</sup> In addition, Shubik cautioned the analyst to concern himself with data validation, sensitivity analysis, aggregation, symmetry and built-in bias. In closing, he stressed that the current tendency in the Defense Department toward "black-box" simulations is dangerous if these are not cross-checked with man-machine games and simple analytical models.

In his paper entitled "Modeling and Markov Processes" (Appendix 6), Matthew Sobel provided a second to Dantzig's offer of mathematical programming as an alternative to simulation. He also observed that previous fears of the size of such programs are no longer generally valid, in that current computers and optimization algorithms can handle very large programs in running times comparable to those of battle simulations. However, to cope with the complexities and dynamic nature of fire support as part of an amphibious landing, he suggests resort to embedding stochastic network decision models within larger simulations. While this is not a totally new idea, it is not often exploited and "the art of such hybrid procedures is primitive and the science is nonexistent". Sobel concluded by citing the need for new developments in four areas:

- a. Optimization program packages for large dynamic decision models.
- b. Synthesis of simulation and network optimization.
- c. Computational solutions of stochastic games.
- d. Decision network representation of the fire support system.

---

<sup>1</sup> It should be recalled, however, that the non-analytic but responsible military decision maker has few tools to check the thoroughness of his "ivory-tower" advisors. To insist that a model have a representation of every facet of actual battle, is expensive and crude but it is an understandable expedient. Ed.

The two remaining papers were concerned with simulation. George Fishman reviewed the characteristics of simulations (Appendix 7), noting six attractive features:

1. Compression of time
2. Expansion of time
3. Model detail
4. Selection of outputs
5. Control of measurement errors
6. Control of variation

In expanding on these points, he stressed the need for weighing the benefits of increasing detail against the costs in model complexity and additional data.

The sixth characteristic of simulation, the control of variation, was discussed in detail. This referred to the ability to control the pattern of variability within a stochastic simulation to reduce the running time required to obtain a given statistical accuracy in the results. Several variance reduction techniques were illustrated which could be important in fire support simulations, through permitting higher resolution at little increase in present running times.

Donald Iglehart, in his paper on "Statistical Analysis of Simulations" (Appendix 8), introduced the regenerative method. Quoting Lieberman's review, "The basis of this method is the collection of data during each of a number of regenerative cycles that will be independent and identically distributed. This requires the existence of regeneration points, which do exist in a wide variety of problems. He described methods for "efficiently" estimating the desired parameters of the simulation with prescribed "accuracy". He also gave two approximation techniques for dealing with non-regenerative systems or regenerative systems for which it is difficult to identify the regeneration points. A major advantage of this technique is the elimination of the need to discover when the system leaves the transient state and enters the steady state. The regeneration method has had important applicability to intermediate size problems (e.g., computer scheduling). Whether or not it will have an impact on fire support simulation models remains to be seen."



## CHAPTER IV

### THE MINI-WORKSHOPS

As noted in the Chapter I discussion of Methods and Techniques, existing models of fire support combine the processes of target generation and target designation. They then model the actual engagement of the target and proceed to the evaluation of the effectiveness of the resulting supporting fires. The subjects of the mini-workshops were drawn from this breakdown; being "Target Generation and Designation", "Target Engagement" and "Mix Effectiveness Analysis". The chairmen of the three working parties each reported on the work of their group and these reports appear as Appendices 9, 10, and 11, respectively. This chapter is a summary of the discussions and findings as they reported them.

Target Generation and Designation (Appendix 9)

The participants in the workshop on "Target Generation and Designation", chaired by Robert Hinckle, again stressed that a dynamic target list was the driving element in fire support studies. They then defined their subject as a "unified process of sensing the presence of enemy units and preparing a ranked, dynamic target array" representing the demands placed on the fire support system, specifically including suppressive, preparation and harassing fires. The process of simulating supporting fires, being dynamic, was visualized as circular. Starting from an ideal list of all possible targets, it moves to an operational target list with both omissions and false targets. In turn, the supporting fires actually delivered and enemy actions both modify the ideal list and the process repeats. A general conclusion of the workshop was that the exact form and level of detail of the process must be tailored to the questions addressed and the decisions to be supported.

Two major areas of inadequacy of present models were discussed. The first of these was the simulation of the operational generation of target lists. The operational basis was taken to be sensor outputs read against a background of prior knowledge and other intelligence. It was generally agreed that sensor performance is well modeled but that the essentially human process of interpreting sensor contacts and, from them, developing the target list, is neither well

understood nor well modeled. For much the same reasons, present models do not adequately represent the generation of requirements for harassment and interdiction; preparation and suppressive fires. Finally, no appropriate way has yet been implemented in machine simulations to represent the human generation of errors in going from the ideal to the actual target lists.

The second area of inadequacy is in representing, in simulations, the dynamic nature of real-world target lists. Operationally, the lists change as the result of new intelligence or sensor contacts, the damage actually inflicted by supporting fires and enemy initiatives. To represent this dynamism, the possible use of a Markov process or stochastic network model is suggested. The state of a system consisting of the ideal target set, the operational target list and fire support assets would be considered at discrete points in time. At a given time, a decision maker allocates his fire resources to targets on the operational list. As a result of the actual implementation of this decision, the system moves, with certain probabilities, to one of a set of possible new states, and the process repeats. The final values of the state variables themselves can be used as the measures of effectiveness. It was emphasized that this concept is a suggestion only and that its efficiency and utility are not established.

The group gave a series of conclusions and recommendations, of which the following is a selection (the complete list is available in Appendix 9):

- Sensor performance models are adequate.
- Ideal target set and operational target list should be kept distinct in the simulation process.
- Dynamic interaction between target set (hence, the target list) and fire plan implementation should be modeled.
- The degree of resolution/aggregation required in the methodology and hence in the target array description is driven by the study question.
- New sensors and data processing systems will place increasing demands on the fire support allocation and engagement process.
- An in-depth study of the target generation process should be made, including command, control and communication; intelligence data processing, and inference.

### Target Engagement (Appendix 10)

The members of the Target Engagement Workshop, chaired by Alan Goettig, set themselves a series of questions and attempted to develop a consensus position on each one that would be helpful to Marine Corps modelers in future studies of fire support. After agreeing that the problem centered on obtaining a low-cost methodology which would treat both an optimum mix of weaponry and a balanced menu of expendable ammunition, they turned their attention to the modeling of suppressive effects of supporting fires.

Their conclusion that suppression and neutralization are the primary things accomplished by real-world fire support threw into sharp relief the fact that current methods deal almost exclusively with the killing or destruction of targets. This, in turn, led to questions of "why?" and "what can be done about it?"

Noting that efforts are under way to seek remedies, the group agreed that the present and basic need is for supportable data on the behavior of troops under fire. Until the difficult task of filling this need has been completed, suppressive effects should be included in models as a set of adjustable parameters. This conclusion was the result of a debate on whether it is more dangerous to use unsupported estimates than to ignore the problem altogether. Including suppression in parameterized form would:

1. Require selection of MCEs that would include it and thereby focus attention on the need for field data and improved models of the effect.
2. Help to identify those cases where the uncertainty about suppressive effects has a significant effect on decisions.
3. Provide a quantitative indication of the need for accurate suppression modeling.

Without wishing to discourage attempts to find alternatives, the group felt that combat experience is the only presently useful source of data and that, of course, it will not be available for the behavior of operators of conceptual future weapon systems. In Appendix 10 will be found references on this subject and a list of individuals working in the area.

This workshop, in addition to tackling their own questions, undertook to comment on the Phase 1 proposals for future development of methodology. After much discussion with an author of the Phase 1 report, the group concluded that they were in general agreement with the near-term proposals as explained verbally. This led to a recommendation that the problem statement and decision process given in the report be redone in a more explicit form.

The far-term recommendation of Phase 1 for development and application of value-driven simulations and algorithmic programs was met with a cautious response. Noting that they appeared not alone in lacking an in-depth understanding of these techniques, the group felt unable to make a strong positive or negative endorsement and urged an exploration and a comparison of them with alternatives prior to making any extensive commitment.

The group did endorse standardizing scenarios (including threats) and concepts of operations as a means of enhancing comparison between studies and reducing study costs. Standardization is not intended to prohibit reasonable excursions, but it might provide some help in the area of target lists for specific, often-occurring combat situations.

No agreement was achieved on how to effect needed improvements in the understanding of the relationships between fire support and rates of advance. The need for more trial data and better modeling of the impact of limited visibility was stressed. This is particularly important in the case of dust smoke or haze over a battle area when guided munitions might be used. The group concluded with a review of the issue raised by Shubik with his phrase "relevance vs realism".

In his personal comments, the chairman noted that the workshop was concerned by the Phase I recommendation for greater detail in fire support modeling. To the group, the acknowledged lack of data indicated, instead, a need for more aggregation in order to speed running times and permit the exploration of the effects of uncertainty in the inputs. In the same vein, it was strongly urged that development of future methodology include the participation of three kinds of people in addition to the technical analysts:

1. Combat experienced personnel who know what kind of data can be obtained.

2. Study managers who have had experience in utilizing methodology to compute answers to someone's questions.

3. Project managers who will have to make decisions on alternative fire support concepts.

Fire Support System Mix Effectiveness Analysis (FSSMEA) (Appendix 11)

This workshop was opened with a review by its chairman, Edward Girard, of two analyses of fire support to illustrate significant improvements still required. He stressed the extreme complexity of tracing the impact of supporting fires through a two-sided, combined arms battle and ultimately developing measures of its effectiveness. The most useful result of such a study was said to be "one that expressed the consequences of the agreed facts and assumptions in the same system of utilities a senior military decision-maker would use if he were actually commanding<sup>1</sup> in the situation visualized. This takes the matter beyond the single point "answer" or "solution" usually produced in a study, which at best has the significance of a revealing example."

Also stressed is that target designations do not come from intelligence and sensor detections alone, but from those inputs considered in the light of the scheme of maneuver which results from the combined arms commander's plan of battle. This places additional strain on the modeling of the target designation process since the commander's function of developing a battle plan must be explicitly represented within or external to any machine simulation.

After the review, this group also took note of the negligible simulation of suppressive effects in existing fire support studies. However, there was seen to be no real barrier to modeling such effects and the work of Lind of

---

<sup>1</sup> Editor's emphasis to call attention to the relevance of this quotation to the "Realism vs Relevance" question and to the earlier recommendation for scenario standardization.

RAND using Marine Corps data from Vietnam was cited as a successful example. For future development, it was suggested that use of algorithmic programming, permitting decomposition of the battle to the level of a single soldier, would, hopefully, allow these effects to flow naturally from the simulation of elementary functional events.

The second major problem discussed was the representation of the combined arms commander in a computer simulation. This requires development of decision algorithms whose output would be "indicative" if not "typical" of decisions taken by senior officers under the conditions portrayed. To develop such algorithms, it was proposed that an advanced man machine model, such as RAC's ADVICE II, be coupled to a high quality combat simulation. A relatively small research team would use this tool to evaluate the candidate algorithms and the successful ones would then be used in the combined arms simulation in which the fire support operations would be embedded. Safeguards would, of course, be retained such as a manual override if the low-level simulations took an absurd course. A manual walk-through would be a useful preliminary to the simulation of novel forces or capabilities.

In considering the six levels of analysis proposed in Phase I, the workshop felt that not all the Marine Corps' decisions in fire support require analysis up to Level 6. They also felt that deficiencies in technique were most serious at Levels 3, 4 and 5. It was accepted that there is little of value to be done to improve the special methods of Level 6 until they can be taxed with an improved quality of analyses and inputs from below. This mini-workshop concluded its report with a proposal for a three-track effort aimed at developing a true combined arms appraisal of fire support:

1. Activate RAC ADVICE II system with the Division Battle Model (DBM)
  - a. Shake-down team, design and start experimental program. 1-2 years at 6 man-year level.

- b. Review Vector DIVOPS and other models as replacements for DBM. 1-2 years at one man-year level.
- 2. Model and Program algorithmic model of physical suppression effects of fire within the rifle company. 2-3 years at 1-2 man-year level.
- 3. Then develop new combat model, possibly improved "ADVICE III" man/machine interface as basis of faster, cheaper, continued experimental programs.

## CHAPTER V

### EDITOR'S OVERVIEW

It is the purpose of this chapter to draw together the separate threads of discussion that were woven through the four-day workshop and to place them in context to see if a pattern emerges which might be even more informative than the sum of the individual recommendations which were made. The starting point for this procedure is the definition of what analysis can be reasonably expected to contribute to the choices the Marine Corps must make with regard to mixes of fire support assets.

#### Expectations vs Reality

We begin at the top. The final stages in a major resource allocation decision in a military service typically consist of meetings attended by two-or-three-star agency chiefs. Since these individuals often represent conflicting interests, their views on the questions in hand will differ. Since they are at a level where surprises in open meeting are inadmissible, they are pre-briefed and arrive with firm positions. Any studies used to support one or the other of the staff recommendations to be discussed will have been reviewed by each busy executive for a total of, at most, an hour or two. If the agency can agree with the study findings, these are generally uncritically accepted. If the "set" of the agency is opposed, the chief will be briefed to attack the scenario, assumptions, input values, rigidity or lack of structure of the methodology or any other "not certain" feature of the analysis. Since the doctrine of "beyond a reasonable doubt" applies in such sessions, the study results are, in the end, left virtually without influence in the choice between the specific options offered for decision. The decision will then be made in a debate which will include factors not part of any formal study, such as the mood of a congressional committee or an inter-agency horse-trade.

If formal studies often have little direct influence at the final decision stage within the service, are they then totally without value?



Absolutely not! They are the best tool known for defining the alternative options or courses of action which are presented to the chiefs for decision; and this is the case for both the lower staff and command levels within the service and in presenting the service's choice as one alternative in the larger decision arenas of DoD and congress.

Some of the consequences of these observed facts-of-life are that (1) no one study should be expected to be decisive in a decision-making exercise; (2) in the results of studies, trade-offs, relationships, and decision-making rules of thumb are of greater and more durable utility than absolute values; and (3) a repeating series of compatible analyses is more useful in providing insight than a number of unrelated, one-shot efforts, however massive they may be. Because the workshop was directed toward the improvement of analytic methodology, these points were not often stressed in the papers or discussion. However, they are germane to the choice of the Marine Corps' future course in dealing with fire support problems. It is therefore important to note the occasions on which they did arise.

In their paper on a "Framework for Effective Fire Support Analysis", Bobick and Peters devoted one whole chapter to the "Fire Support Decision Interface". In the first paragraph of that chapter (Appendix 2, page 6), they make a blunt but salutary observation; "Specifically, Marine Corps Fire Support analyses have failed principally through the oversight of trying to replace rather than augment the decision process of the decision-maker." They continue, "It must be emphasized that the analysis should be used to clarify the decision parameters and illuminate alternative courses of action,..." In short, the Corps' disappointment with their fire support studies is said to stem less from inadequacies in the state-of-the-art (although there are plenty of those!) than from unrealistic expectations of what any study can achieve. This results when the actual impact of study findings is compared to a goal which is impossible, both in practice and in theory. Lessons drawn for the future development of fire support studies should take full account of this experience.

The place of a study in the decision-making environment is addressed from a somewhat different viewpoint by Shubik in his section on "The Purpose and Process" (see Appendix 5). To ensure the utility of analytic results, he suggests that it is of great importance to do a study of the implementation process from the inception of a projected analysis to how it will finally influence a weapons mix and force structure problem. Rotations of military personnel can leave to an analytic team much responsibility that should remain with a sponsor. And in addition to this reversal of roles, there is the confusing fact that "even if all individuals at different points in an organization are efficient and rational, the overall performance of the organization need not be rational or even reasonable. In terms of studies such as detailed fire support simulations, overall organizational rationality must force us to ask how do they fit in, in importance, for the overall organization?" All of this suggests that in the Phase III review of constraints in the Marine Corps' mechanisms of decision-making, it is necessary to look at organizational and subjective questions, as well as objective factors such as the limitation on study resources. It also means that the results of a broadened Phase III should be a major input to the improvement in the definition of the questions to be answered by fire support analyses, which was recommended in the reports on Phase I.

#### Validity

While, as noted above, absolute results of fire support studies cannot be made proof against "reasonable doubt", the models and inputs must be validated whenever possible. This need for continual attention to proof and verification is not unique to military analytical studies. The theories and even the laws of physics are also mathematical models subject to test. However, in the world of physics, the standards of validation are far higher. No theory is accepted for application until it is shown to describe adequately all relevant, previously known phenomena and to predict accurately the results of experiments yet to be performed. In checking theories, great attention is, of course, paid to the precision of experimental results and to the purity and completeness of the data and the model in which they are used. The risks of error, without such rigorous testing, reduce the unvalidated model to the status of so many exotic symbols on paper.

Military analysts pay lip service to these same standards of validation, precision of input, completeness of experiment and model, and to the testing of the sensitivity of results to variations in input. Several workshop participants discussed these requirements on analyses and most have added that these requirements are almost never met. Yet, to an astonishing degree, the discussion then proceeds as though they were! This clearly puts the buyers of large modeling efforts in a dilemma and, to some extent, places analysis on the plane of some aspects of the practice of medicine. Much practical help may be rendered the client, but it may never be completely clear how the result was achieved or what side effects will nullify the next attempt. It also places a premium on obtaining the services of the most skilled practitioners available.

Ideally, to improve the situation, there are two courses of action needed, one for the analysts and one for their clients. Analysts should teach potential clients the importance of validation of model and input as well as the need for incorporation of sensitivity testing in the total study. They should then develop a design of the total program which includes an effort balanced among modeling, validation and the study of decision questions. The client can participate in an implementation study and aid in defining the decision questions so that the balance can be struck. Once a sound program is established, the client can take the unpopular step of funding an effort in which the visible size (and glamor) of the model and the volume of study output are reduced in favor of increasing the assurance of validity in the results.

But the reason validation and sensitivity testing are often neglected is that they are extremely difficult to do and the attempt is costly to carry out. What are the practical steps that can be added to an analysis program to ease these problems?

Possible courses of action, largely drawn from workshop papers and discussion, include the following:

First, change the goal. Give up any desire for a sausage machine which delivers incontrovertible decisions. Adopt a research attitude of continually seeking to improve, by any means available, the understanding of the operational processes and interactions of fire support. Plan for the

integration of modeling, gaming and computer simulation with field trials, historical research and the use of Electronic Warfare, aircraft and other hardware simulators. Put operators, study managers and decision makers on the methodology development team and arrange for the team to spend time observing field trials and exercises. Seek inter-relationships between elements of the fire support system, synergisms and trade-offs, all of which are of more use than single-point, absolute results when fitting systems costs into a shifting budget.

Second, control inputs to models, accepting only those for which values are or can be made available from trials, combat data or other valid external source. If a function or process must be "parameterized", as was recommended for the modeling of suppressive effects, make the appropriate sensitivity testing a built-in part of the model. Since it is virtually impossible to validate a complete model of a large military action, at least schedule the validation of sub-models such as the functioning of, say, a fire support coordination center. Finally, build into the analysis program statistical investigation of variation of results due both to the stochastic nature of the operations modeled and to the broad confidence intervals typically associated with the inputs.

Third, schedule a substantial fraction of the study effort for the interpretation of results and the design of decision aids and graphics for presentations to senior decision makers. The further the results are from single-point, absolute value allocation recommendations, the harder it is to extract and convey the lessons learned from the study, or better yet, a series of studies. On the other hand, such general lessons are not easily dismissed by debating tactics and retain their influence at higher organizational levels.

The above thoughts on future study efforts are largely directed toward the management approach. What positions did the workshop participants take on new methodologies?

### New Techniques

The keynote on innovations was one of caution. A series of techniques were discussed by experts in their development and application. Each presenter gave a review of the strengths and weaknesses of his technique and concluded by saying, with greater or less certainty, that it should be applicable to the Marine Corps' Fire Support Problem. The reviewers and mini-workshops then pointed out that validity, practicality, comparative utility or other factors relating to the choice of any of the techniques, were not yet established. Such comments were even applied to some of the methods discussed by Cherry, which are already incorporated in a series of large models of theater or divisional combat!

The caution of the professionals suggests that in attempting to adapt new techniques to their problems, the Marine Corps has two, almost equally unpalatable options. One is to go directly for new models of fire support which incorporate the innovations and, in doing so, to accept a high risk of achieving little at great cost. The second is to undertake a program of fundamental research in methodology before proceeding to further work on fire support requirements. Both of these options tend to encourage a search for a middle ground.

### Old Problems

In terms of major unsolved modeling problems, the workshop developed little that was new. Continued research was the only answer offered to the problem of modeling suppressive effects, with a parametric treatment incorporated in fire support studies until a definitive model was available. Opinion remained divided on how to make a dynamic target list available to the battle simulation. It was agreed that such a list was vital, that it had to allow adaptive behavior by the enemy, and that it had to reflect the accumulating effects of prior engagements by supporting fires. But recommendations were made both for supplying the list from outside the simulation and for modeling the target generation and designation process within the simulation.

Finally, the need for evaluation of fire support mixes in the context of a combined arms battle was stressed, but there was no method offered for simulating the planning and decision functions of the commander. Again, the establishment of research game was recommended, using ADVICE II, to try to develop decision algorithms to be incorporated in a free-running simulation.

### Conclusion

The FSMRS was carefully structured in phases. Phase I was to review existing methodologies and models and to develop a tentative course of future model development. Phase II has been a workshop of experts reviewing new methodologies and the recommendations of Phase I. Phase III is a discussion of the decision making process to be supported. The record of Phase II suggests that the development of future fire support study programs should, as planned, follow a similar course:

1. Define the impact which studies can reasonably be expected to have and the specific decision questions to be addressed. (Recall that no one believed that a single model could meet all of the requirements.)
2. Develop models tailored to the questions, paying careful attention to the validation of sub-models and validity of inputs; use statistical techniques to determine the significance of results.
3. Devote a significant portion of the study effort to the analysis of results, extraction of lessons and development of decision aids.

In all of this, the emphasis shifts away from methodological development and toward a strong and careful effort on step 1; the use of simpler models which are more commensurate with their degree of validation and our poor knowledge of many inputs; and many replays and variations of studies to gain insight into the fire support process and to control statistical variability.

Final Note

The rigorous critique of analytical methods which characterized this workshop might suggest to some that technical analysis is not worth the obvious cost and effort. Let anyone who finds his thoughts moving in that direction, apply the same standards of criticism to the process of making decisions only by debate, based on unsupported gut feelings!

## REFERENCES

1. "Fire Support Requirements Methodology Study": Verbal and Mathematical Description of the Marine Corps Fire Support System, Potomac General Research Group, Contract M00027-74-A-0042, (April 1975), UNCLASSIFIED
2. "Fire Support Requirements Methodology Study": Review of Theories and Techniques Applicable to Marine Corps Fire Support, Potomac General Research Group, Contract M00027-74-A-0042, (June 1975), UNCLASSIFIED
3. "Fire Support Requirements Methodology Study": Survey of Programmed and Operational Models for Evaluating the Fire Support System, Potomac General Research Group, Contract M00027-74-A-0042 (July 1975), UNCLASSIFIED
4. "Fire Support Requirements Methodology Study": Phase I Summary Report, Potomac General Research Group, Contract M00027-74-A-0042 (July 1975), UNCLASSIFIED



APPENDIX 1

FIRE SUPPORT METHODOLOGY WORKSHOP

ATTENDEES, PROGRAM AND  
WORKSHOP PARTICIPANTS

Fire Support Methodology Workshop Attendees

Dr. Robert D. Arnold, Ketrion, Inc., Workshop Manager  
John Bobick, Stanford Research Institute  
LCOL Gene E. Brennan, HQ, U.S. Marine Corps  
Dr. Kwai-Cheung Chan, Institute for Defense Analyses  
LCOL Richard Chenault, Marine Corps Tactical Systems Support Activity  
Dr. Peter Cherry, Vector  
George B. Dantzig, Stanford University  
Robert V. Dennis, Potomac General Research Group  
Robert H. Dickman, Office of Naval Research  
Leon Feldman, Center for Naval Analyses  
George S. Fishman, University of North Carolina  
Edward Girard, Potomac General Research Group  
Alan H. Goettig, Naval Weapons Center, China Lake  
LCOL Kenneth P. Harrison, Marine Corps Development & Education Command  
CAPT Fred Hartman, U.S. Army Concepts Analysis Agency  
Dr. Robert Hinkle, Naval Surface Weapons Center, Dahlgren  
Timothy J. Horrigan, Horrigan Associates  
Donald Iglehart, Stanford University  
Professor Gerald J. Lieberman, Stanford University  
Jack Lind, The Rand Corporation  
Rufus C. Ling, U.S. Army Concepts Analysis Agency  
Lawrence J. Low, Stanford Research Institute

Fire Support Methodology Workshop Attendees

(continued)

Ronald Magee, U.S. Army Combined Arms Combat Developments Activity

Ronald New, Catholic University

Dr. D. B. Osteyee, Office of Naval Research, Pasadena

Lloyd S. Peters, Stanford Research Institute

COL G. H. Polakoff, Marine Corps Dev. & Education Command

George E. Pugh, General Research Corporation

Edward K. Reedy, Georgia Institute of Technology

LCOL R. S. Robertson, HQ, U.S. Marine Corps

Dr. Robert Ryan, Office of Naval Research

MAJ Ludwig J. Schumacher, Marine Corps Dev. & Education Command

Professor Martin Shubik, Yale University

J. Randolph Simpson, Office of Naval Research

James G. Smith, Office of Naval Research

Professor Matthew J. Sobel, Yale University

Professor James Taylor, Naval Postgraduate School

Dr. Martin Tolcott, Office of Naval Research

Thomas C. Varley, Office of Naval Research

MAJ Robert L. Vogt, Marine Corps Development & Education Command

Richard E. Zimmerman, Potomac General Research Group

PROGRAM  
FIRE SUPPORT METHODOLOGY WORKSHOP

U.S. Naval Postgraduate School

Monterey, California

4-7 August 1975

Monday - 4 August

Chairman: T. Varley, Office of Naval Research

Room: Ingersoll Hall 122

8:00 - 8:30	Registration
8:30 - 9:00	Welcome and Announcements
9:00 - 10:00	The Fire Support System and its Problems K.P. Harrison, MCDEC, and R.E. Zimmerman, PGRG
10:00 - 10:20	Coffee Break
10:20 - 11:20	Summary of Models and Techniques R.V. Dennis, PGRG
11:20 - 12:00	Dynamic Planning R. New, Catholic University
12:00 - 1:30	Lunch
1:30 - 2:30	Value-Driven Simulation G. Pugh, General Research Corp.
2:30 - 3:00	Coffee Break
3:00 - 4:00	Algorithmic Models T. Horrigan, Horrigan Associates
4:00 - 4:45	Tentative Development Plan R.E. Zimmerman, PGRG, and L.J. Schumacher, MCDEC
6:00 - 8:00	No-Host Cocktail Party (Commissioned Officers and Faculty Club)

Tuesday - 5 August

Chairman: J.R. Simpson, Office of Naval Research  
Room: Ingersoll Hall 122

8:50 - 9:30	Concepts for an Overall Fire Support Analysis Methodology John Bobick, Stanford Research Institute
9:30 - 10:30	Mathematical Programming and Its Role in Fire Support George Dantzig, Stanford University
10:30 - 10:45	Coffee Break
10:45 - 11:45	The Role and Use of Lanchester-Type Models Peter Cherry, Vector Research, Inc.
11:45 - 12:45	Uses of Game Theory, Gaming, and Model Building in the Study of Fire Support Problems Martin Shubik, Yale University
12:45 - 1:45	Lunch
1:45 - 2:45	Methods of Simulation Analysis George Fishman, University of North Carolina
2:45 - 3:45	Modeling and Markov Processes
3:45 - 4:00	Coffee Break
4:00 - 5:00	Simulation and Statistical Inference Donald Iglehart, Stanford University

Wednesday - 6 August

Chairman: R. Dickman, Office of Naval Research  
Rooms: Ingersoll Hall (Room numbers to be announced  
at meeting.)

Three miniworkshops will be conducted in parallel. Each attendee will select one on Tuesday afternoon at the registration table. In case of maldistribution, some attendees may not receive their first selection.

8:30	Target Generation and Designation <u>Session Chairman:</u> Robert Hinkle, Naval Surface Weapons Center, Dahlgren
8:30	Target Engagement <u>Session Chairman:</u> Alan Goettig, Naval Weapons Center, China Lake
8:30	Fire Support System Effectiveness <u>Session Chairman:</u> Ed Cirard, PGRG

Thursday - 7 August

Chairman: J. Smith, Office of Naval Research

Room: Ingersoll Hall 271

8:30 - 10:10	Reports of Miniworkshop Chairmen
10:10 - 10:30	Coffee Break
10:30 - 12:00	Summary of Invited Papers G. Lieberman, Stanford University, and J. Taylor, Naval Postgraduate School
12:00	Workshop Adjourns

## MINI-WORKSHOP PARTICIPANTS

### Target Generation and Designation

Robert Hinkle - Chairman

Peter Cherry

Robert Dennis

Leon Feldman

Edward Reedy

Ludwig Schumacher

James Smith

Matthew Sobel

Robert Vogt

### Target Engagement

Alan Goettig - Chairman

John Bobick

Kwai-Cheung Chan

George Fishman

Kenneth Harrison

Fred Hartman

D. B. Osteyee

Lloyd Peters

James Taylor

Richard Zimmerman

MINI-WORKSHOP PARTICIPANTS  
(continued)

Fire Support System Mix

Edward Girard -- Chairman

Gene Brennan

Richard Chenault

Timothy Horrigan

Donald Iglehart

Gerald Lieberman

Jack Lind

Rufus Ling

Ronald New

R. S. Robertson

Martin Shubik

Randolph Simpson



APPENDIX 2

FRAMEWORK FOR EFFECTIVE FIRE SUPPORT ANALYSIS

# FRAMEWORK FOR EFFECTIVE FIRE SUPPORT ANALYSIS

by J.C. Bobick and L.S. Peters

## I INTRODUCTION

### A. Definition of Fire Support

The term "fire support" embraces the employment of a variety of weapons to provide supporting fire to ground units in combat. A more precise and relevant term to describe these weapons is "supporting arms," defined<sup>1</sup> as air, naval, and artillery weapons of all types when they are employed to provide support fire for ground units. It should be observed that the term fire support denotes specifically a function rather than a weapons system. The weapons, when used as supporting arms, fulfill the function of fire support.

The three general types of supporting arms employed in the fire support role may be more definitively described as follows:

- (1) Air--The air weapon system consists of the carrier or land base, the aircraft, the aircraft weapon delivery subsystem, and the ordnance.
- (2) Naval Shorefire--The naval combat ship weapon system consists of the ship platform, the gun/missile mount(s) and associated fire control subsystem, and the ammunition.
- (3) Artillery--The artillery weapon system consists of the gun, howitzer, missile launcher, mortar, associated fire control subsystem, and the ammunition.

The Marine Corps has expanded the scope of the term "supporting arms" to include two additional types of weapon systems: armored combat and special purpose systems.<sup>2</sup> The armored combat weapons system includes tanks and armored amphibians that mount gun systems; the special purpose weapon systems include area denial weapons, weapons employed in psychological operations, and riot control weapons. Thus, the term supporting arms embraces all methods of neutralizing and destroying designated ground targets in support of Marine landing forces in amphibious assault and sustained combat operations ashore. The only Marine Corps weapons excluded from the scope of the term are the individual and man-portable weapons of infantry systems, antiair warfare weapons, and nuclear weapons.

The fire support function is subdivided with respect to the proximity of the target to friendly force. These subdivisions are:

- Close Supporting Fire, defined<sup>1</sup> as fire placed on enemy troops, weapons, or positions which, because of their proximity, present the most immediate and serious threat to the supported unit.
- Deep Supporting Fire, defined<sup>1</sup> as fire directed on objectives not in the immediate vicinity of our forces, for neutralizing and destroying enemy reserves and weapons, and interfering with enemy command, supply, communications, and observations.

The delimitation of close fire support is significant because it introduces coordination, control, and timing problems not involved in deep support. This is especially true of close air fire support, which by definition requires detailed integration of each mission with the fire and movement of the friendly forces on the ground.

#### B. The Fire Support Problem

The description of the "Fire Support Problem" varies with the different aspects of fire support with which people are involved. The roles people play in fire support include those of planner, technician, strategist, tactician, requestor, allocator, server, and supporter.

The planner is concerned with decisions regarding research, development, test, evaluation, and procurement of fire support equipment. The technician is concerned with the application of technologies in the design of weapons systems. Strategists include the amphibious force commander who is interested in establishing a strategy appropriate to the contingency objective which incorporates the fire support with the other elements of the amphibious force. Tacticians are concerned at the air wing, artillery battalion, or naval shorefire support ship task level with establishing the most efficient tactical scheme of employing the available resources. The requestor (i.e., the ground combat force) is concerned with getting sufficient amounts of fire support to accomplish his mission objective. The allocator is concerned with appropriately selecting among the resources that can be placed at his disposal. The server (i.e., the aircraft pilot, battery commander, or ship shorefire officer) is concerned with the successful completion of the assigned task. At the maintenance support level the

concern is to maintain the consumables, ammunition ready supply, fuel, spare parts for maintaining the weapons, and the necessary logistic system that permits this end.

Regardless of the roles of people involved in fire support, a common point on which all agree is that fire support is vitally important in the conduct of assault and defensive operations, particularly those associated with amphibious warfare. This high level of importance supports the expenditure of substantial effort in the analysis of requirements, performance, utilization, and effectiveness of alternative fire support systems. The overall objective of such analysis is to support decisions that must be made in seeking the best system achievable within real world constraints. Such real world constraints include technological limits, budgets, manpower, logistics, and political considerations.

#### C. Scope of this Paper

In order to perform effective fire support analysis, a framework needs to be established to provide the means of acquiring valid inputs to the analyses, utilizing the proper tools for the analyses, and using the results of the analyses to update the common base of knowledge on fire support. The framework depicted in Figure 1 is proposed to fulfill this need; it is the focus of this paper.

As indicated in Figure 1, there are three basic components to the proposed framework; namely, the decision interface, the quantitative analysis, and the data reservoir. Each of these components will be discussed in detail in this paper. The decision interface component is discussed first. The analysis support required by various decisionmakers and the associated measures of effectiveness and decision aids are described. Secondly, quantitative analysis of fire support systems is considered. A system formalization of the physical fire support system which is amenable to analysis is derived, and the key elements of quantitative fire support analysis are described. Fire support analyses are categorized into three levels, namely force level requirements, operational concepts, and technology assessment analyses. The analysis tools, models, and techniques useful for these

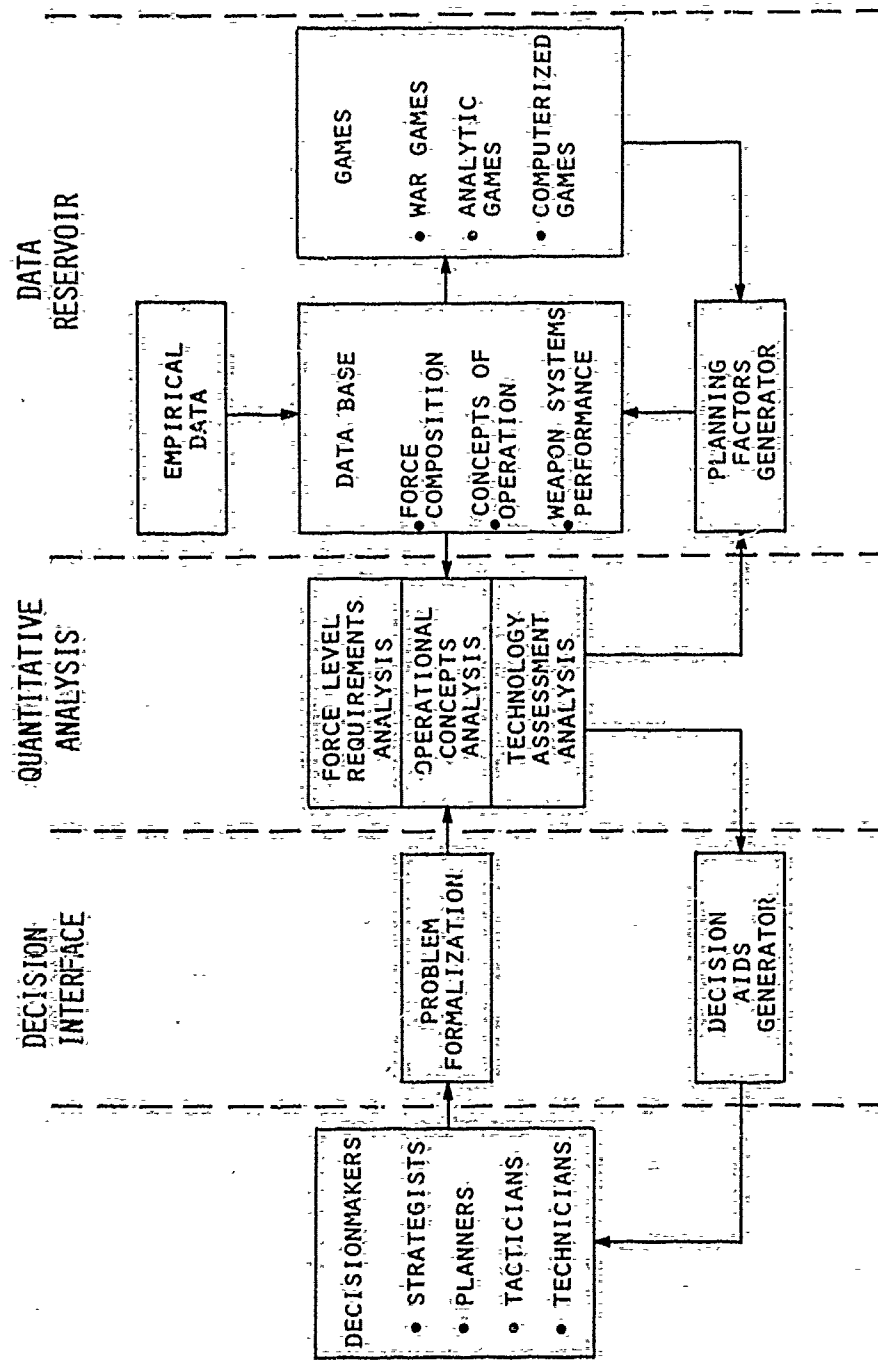


FIGURE 1 FRAMEWORK FOR EFFECTIVE FIRE SUPPORT ANALYSIS

various analyses are described. Thirdly, the role of the data reservoir, in fire support analysis, the type of data in the data reservoir, and the sources of this data are discussed.

## II FIRE SUPPORT DECISION INTERFACE

Modeling of large scale military systems has become an integral part of many analyses of force level requirements, force operations, and technology applications. The utility of these models in the decision process and the relationship between model builders and decisionmakers have recently been the subject of criticism.<sup>3</sup> Existing fire support analyses for the Marine Corps bear out much of such criticism. Specifically, Marine Corps Fire Support analyses have failed principally through the oversight of trying to replace rather than augment the decision process of the decisionmaker. It must be emphasized that the analysis should be used to clarify the decision parameters and illuminate alternative courses of action, not to replace the decisionmaker or his responsibility.

Thus, as indicated in Figure 1, the first component of the framework for effective fire support analysis is the interface between the analyst and the decisionmaker. Prior to employing models to perform quantitative analysis, it is essential to translate the fire support problem, as conceived by the decisionmaker, into a formalization which is amenable to analysis. This formalization of the problem requires an appreciation of the perspective of the decisionmaker, which can be attained only by communication with him. Measures of effectiveness of alternative systems need to be identified during the intercourse. Also, decision aids which will facilitate the comparison of alternative systems in view of both quantitative and qualitative considerations need to be devised. Neglect of sufficient interface between the analyst and the decisionmaker will doom the analysis to failure.

The character of the analysis support required by a decisionmaker is dependent upon the type of decisions with which he is concerned. It is therefore useful to categorize decisionmakers according to their concerns and identify the associated measures of effectiveness and decision aids.

#### A. Hierarchy of Decisionmakers

The highest level of decisionmakers is concerned with inter-system force level requirements. Strategists and planners are concerned with the adequacy of planned resources to carry out the assigned role in national defense, e.g., planning the necessary naval shorefire support and naval aviation for the mission of projection of sea power ashore. The type of decisions include the establishment of policy and setting of organizational responsibility for the provision of fire support capability for amphibious warfare.

At the intra-system force level requirements level, planners are concerned with obtaining balance within supporting arm systems (air, naval shorefire support, or artillery). The type of decisions concern the establishment of emphasis, such as, fighter, attack, or support aircraft in obtaining the balance that best counters the threat characteristics.

At the operational concepts decision level, tacticians, requestors, allocators, servers, and supporters are concerned with tactical and operational considerations regarding weapons systems at their disposal. The type of decisions at this level include establishing modes of operation, schemes for allocating resources among competing demands, and providing maximum weapon system availability.

At the technology assessment level, technicians and R&D planners are concerned with the pursuit of technologies that will reduce constraints on amphibious warfare. The type of decision involved concerns the balancing of development risk with the potential payoffs in terms of improved tactics and weapon system performance.

The types of decisions addressed at the various levels have unique characteristics. Therefore, the choice of tools and techniques used to provide analytic support must be tailored to the particular decision level of concern.

#### B. Measures of Effectiveness

In performing fire support analysis it is necessary to identify Measures of Effectiveness (MOEs) which adequately reflect the effectiveness



of alternative systems to the decisionmaker. Such MOEs must be directly related to the objectives of the fire support system and be analytically tractable or measureable. In the assessment of the fire support system, a parallel hierarchy of MOEs must be developed that correspond to the decision levels identified in the prior section. Following is a discussion of such a correlation; this discussion is not intended to be complete, but, rather, to indicate the relative nature of MOEs at the various decision levels.

At the highest decision level (inter-system force level requirements) the MOEs are the most encompassing and as a result the most abstract relative to an individual fire support weapon. At this level of decision, measures such as the amount of ground gained or lost as a function of time, casualties inflicted in ratio to casualties taken, and the expected outcome of the battle are most meaningfully compared with planning objectives. With MOEs of this type produced for each alternative fire support system, it is possible for the decisionmaker to select a preference, and resolve issues such as the relative emphasis on air, artillery, and naval shorefire support.

For intra-system force level requirements decisions, the MOEs are less encompassing and more easily correlated with operational understanding of weapon systems employment. At this level such measures as the number of military mission objectives achieved and responsiveness attributes could be meaningfully interpreted as they relate to the system mix attributes. These measures for reasonable alternative weapon system mixes make it possible for the decisionmaker to establish system balance and to select a preference that minimizes resources while meeting the higher decision level requirements.

At the next successive decision level, operational concepts, the MOEs are most closely related to measures used in the management of system operations. For example, such measures as the accomplishment per weapon operation (e.g., target kills/sortie for aircraft) and the continuity of fire support to the supported forces are most meaningful. These measures permit the expression of effectiveness sensitivity to alternative operating tactics and provide for the selection of tactical preference within the context of balanced system capability previously mentioned.

At the technology assessment decision level, the MOEs are most closely related to performance measures. Such measures as the probability of acquiring a target, CEP of target location designation, probability of target destruction, and other similar MOEs are most valuable to the technology oriented decisionmaker.

The prior discussion has focused on only the effectiveness measures for the fire support system; it is recognized that costs and other constraining aspects need to be considered. Cost-effectiveness techniques in which a ratio is used to permit the direct comparison of alternatives are commonly used. In addition to the cost constraints, future decisions will increasingly have to consider constraints of other resources such as energy, amphibious lift, and personnel.

#### C. Decision Aids

The models used in fire support analysis serve to produce data concerning the expected effectiveness of alternative weapon systems, operating modes, or technological innovations. Presenting these data to the decisionmakers in its raw form or only in terms of MOEs is neither an effective, fast, nor practical solution. The resulting data from the models will have to be reduced by methods of graphical analysis or incorporated into a decision analysis format before presentation for decision. Since both of these analysis techniques are widely known,<sup>4,5</sup> the specifics of the decision aids will not be discussed here.

It is important to make the point that in the pursuit of a fire support analysis methodology, effort should be directed to the formatting of results for the decision process. In the absence of such effort, the true value of the fire support analysis methods may never be realized.

### III QUANTITATIVE FIRE SUPPORT ANALYSIS

#### A. System Description of Fire Support

As indicated in Figure 1, the second component in the framework for effective fire support analysis is quantitative analysis. In order to perform such analysis, the physical fire support system must be represented by a system description which captures the essence of the physical system in an analytically tractable form. Such a system description, based on the interaction of the physical fire support system with the enemy target array, is conceptualized in Figure 2.

The surveillance process is peripheral to fire support, yet intimately related. The purpose of surveillance is to sense the presence of the opposing force target array. The information gathered via the surveillance process is refined in the target acquisition process in which specific targets are detected, identified, and located in sufficient detail to permit the effective employment of weapons. Also, the decisions about which targets require fire support and the priority order in which these targets should be addressed (i.e., target analysis) can be included. The output of the target acquisition process is therefore a target list that includes the target identification, location, effect desired, and the fire support weapon preference.

The target allocation process matches the specific type weapon (including ordnance and ordnance configuration) with the target within its environment. This process accounts for the weapons, their current and projected availability status, the availability of logistics, and other factors that could affect the allocation and expenditure of fire support resources.

The weapon delivery process includes all necessary operations of the weapon systems. This includes all preparation of the fire delivery means and necessary ordnance, coordination, execution of the fire mission, and return of the weapon system to a state of readiness.

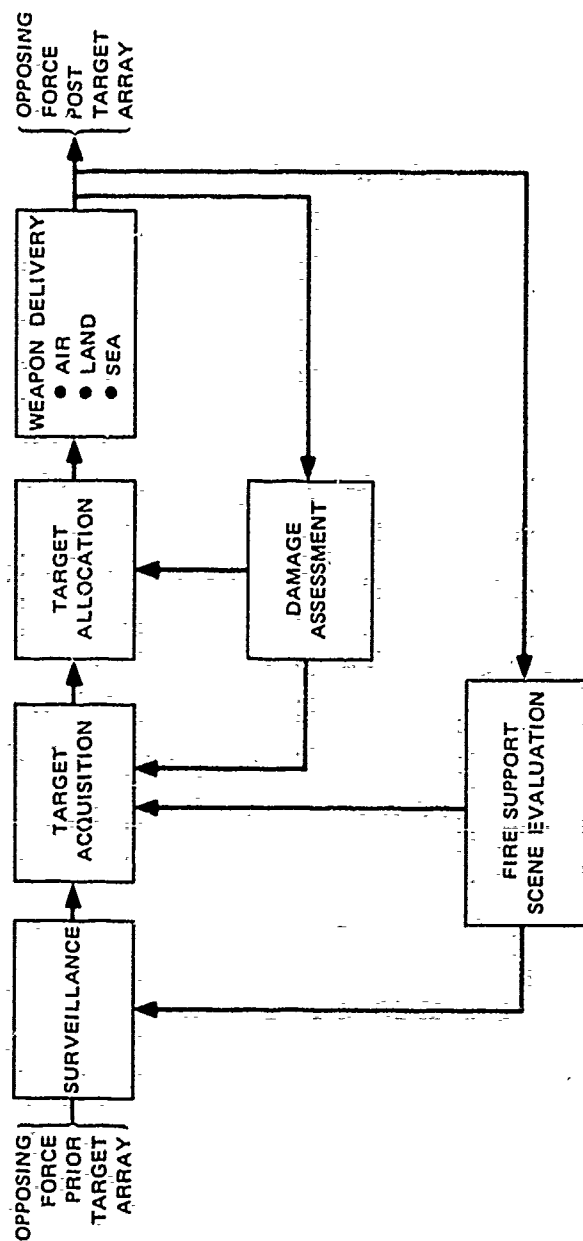


FIGURE 2 CONCEPTUAL FIRE SUPPORT SYSTEM

The damage assessment process is initiated with the delivery of ordnance on the target. The assessment of the results is necessary to indicate when the desired effects have been achieved, to determine changes in the target status, and to update the changes in the target acquisition information. This assessment provides the essential feedback to improve both the estimates made about fire support weapon system requirements and the detailed state of knowledge about the target array resulting from the application of the supporting firepower.

The last process is fire support scene evaluation. The effects of the application of the fire support are assessed for revisions of the opposing force target array in terms of composition and disposition. This assessment serves to update the target acquisition and overall surveillance information which, in turn, impacts on planning for the conduct of friendly operations.

It should be noted that the input and output of the conceptual fire support system of Figure 2 are the opposing force target arrays. The differentiation into prior and post target arrays signifies that the primary objective of fire support is to achieve a modification of the enemy target array. Any measure of the effectiveness of fire support system must represent the capability of a candidate system to make such modifications. This implies a change in target array with time. More specifically, the application of fire support serves to modify an enemy target where the desired modifications are specified in terms of desired effects and a schedule for their accomplishment.

#### B. Elements of Quantitative Fire Support Analysis

In general, quantitative fire support analysis involves several key analysis elements. These elements, as illustrated in Figure 3 in the heavy-lined boxes, include:

- Fire Mission Generation
- Fire Mission Allocation
- Fire Support System Effectiveness Analysis
- Fire Support System Cost Analysis
- Fire Support System Preference Selection

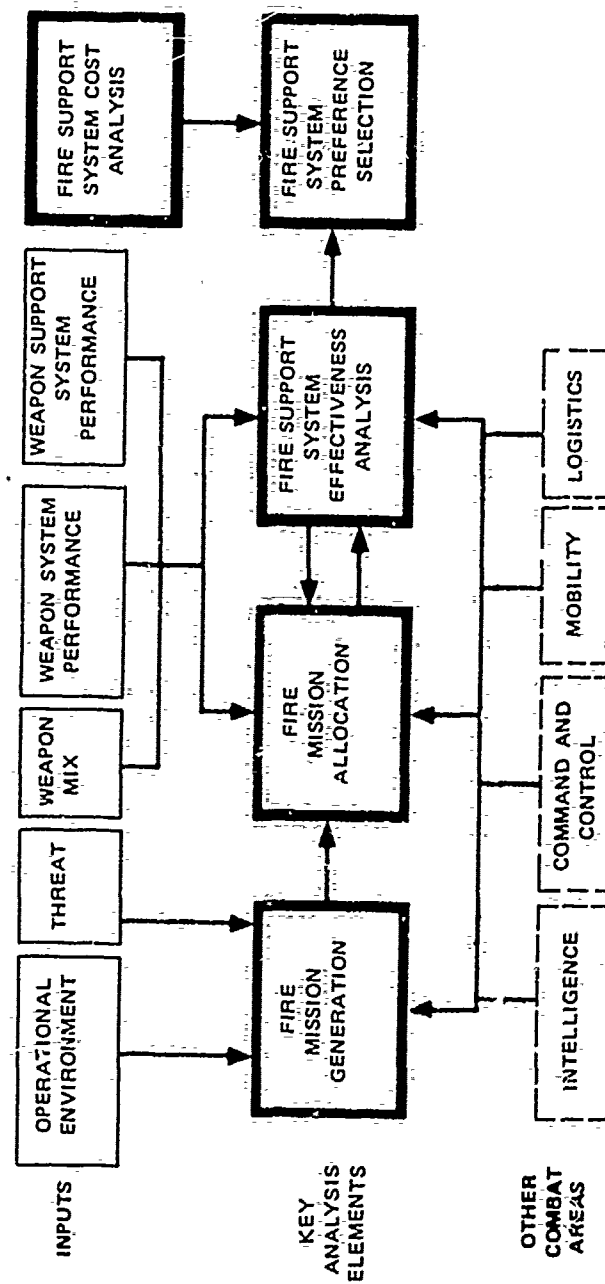


FIGURE 3 INTERRELATIONSHIPS OF KEY ELEMENTS OF FIRE SUPPORT QUANTITATIVE ANALYSIS

Also shown in Figure 3 are the relationships of the key inputs and the combat areas other than firepower to these key analysis elements. The details of carrying out these analysis elements depends upon the particular concerns of the decisionmaker for whom the analysis is being undertaken. However, it is useful to discuss the general character of these analysis elements.

Fire support exists to achieve desirable modifications of the hostile target array. Thus, to quantitatively analyze the degree to which a fire support system meets this objective, credible targets and missions must be generated to represent the demand placed on the fire support system. The missions (or demands) for fire support must reflect the operational environment and the threat posed by enemy forces. Possible operational environments cover a wide spectrum of conflict situations, geographical locales, friendly missions, operational concepts, and force levels. The threat describes the opposition, including order of battle information, general characteristics of the hostile force, the disposition of the enemy's available forces, and the enemy's tactical doctrine.

Fire mission generation involves several tasks. These include generation of the array of actual targets, the acquisition of targets, and the synthesis of a target/mission list. The actual target array describes the hostile forces in terms of types, size, location, and activity. This array is a dynamic entity and changes as the battle progresses. For the purposes of a specific fire support analysis, at any instant in time this array represents exactly where the enemy is and what he is doing.

Target acquisition is accomplished by surveillance, reconnaissance, and other target acquisition systems. These systems act on the array of actual targets and produce the material for the mission list. The friendly forces will not know the exact location and description of each hostile target. The system used to acquire targets will not only miss some targets completely, mislocate or misdescribe others, but undoubtedly will introduce spurious ones.

The result of the target acquisition system acting on the actual target array is a picture of the opposing forces as seen by friendly forces, which is in effect the acquired target array. After analyzing

the possible impact of these targets upon the friendly scheme of maneuvers and missions, they are placed into a priority listing. This list is designated a target/mission list. The word "mission" is included in this term because the list contains fire missions such as illumination, harassment, and interdiction, as well as destruction and neutralization. Also, based on the target description, location, and effect desired, a supporting arms preference is indicated.\* This list now forms the demand for the use of fire support systems.

The second key analysis element is fire mission allocation. Having established the demand for fire support, an attempt is made to allocate resources to fulfill this demand. The allocation function addresses the manner in which missions are actually assigned to weapons. Factors involved in this process relate to the target, such as type, location, priority, and duration; to the supporting arm, such as capability, availability, and responsiveness; and to constraints, such as logistic supportability, safety of friendly troops, desire for surprise, obstacle creation, and civilian casualties.

Necessary inputs to this allocation process are the available weapon mix, the weapon system performance characteristics, and the weapon support system performance characteristics. The weapon mix, that is, the types and numbers of weapons to be considered, must be available to start the allocation process. The weapon performance characteristics of the individual weapon system under consideration must be input since they influence the preferred usage of the weapons. Similarly, the performance characteristics of the support system affect such factors as ordnance availability, resupply rate, and rearming and refueling time which, in turn, affect weapon system availability.

The third key analysis element is fire support system effectiveness analysis. Once a mission has been allocated to a fire support means, the

---

\* It should be noted that neutralization of some targets may be achieved without placing fire upon them. For example, some targets may be neutralized and thus prevented from accomplishing their objective, by employing ECM. In this use ECM may be considered as a weapon substitute, and certainly its use should be integrated with conventional fire support means.



next function to be performed entails the analysis of how effectively the assigned mission is accomplished. The particular measures of effectiveness (MOEs) generated in this step in the analysis <sup>are</sup> ~~is~~ dependent upon the scope of a particular study and the decision level at which the quantitative results will be used. Obviously, the weapon mix, weapon system performance, and weapon support system performance inputs bear directly upon the fire support system effectiveness analysis.

The fourth key analysis element is fire support system cost analysis. In days of great emphasis on achieving the maximum use of resources, the cost of resources used in providing fire support must be considered in selecting among alternative weapon systems. Because quite disparate systems are necessary to provide fire support (i.e., ground, sea, and air systems, which operate in different environments and differ as to whether fire support may be the sole reason for their existence), it is particularly important that a consistent, overt method of costing be used and that the method of allocating costs to multimission systems be credible. In particular, life-cycle costs should generally be considered in cost-effectiveness evaluation.

The final key analysis element is fire support system preference selection. Key ingredients to the system preference selection are the results of the effectiveness analysis and the cost analysis. These must be related in some way to show how the costs vary with effectiveness. A criterion or criteria must be established to enable selection among alternative systems. The most common method is to select a fixed level of effectiveness and determine which system can provide that level of effectiveness at least cost or, alternatively, to establish a fixed cost (budget) and determine which system provides the greatest level of effectiveness for that budget.

The objective of the system preference selection is to provide the decisionmaker with a set of promising alternatives based on the information and constraints considered in the analysis. Because many constraints (e.g., political constraints) are not quantifiable and require qualitative judgements by a decisionmaker, a quantitative analysis must present not "the answer" but several promising alternatives.

As indicated in Figure 3, there exist relationships between the combat area of firepower, which is of major concern in a fire support analysis, and the other combat areas; namely, intelligence; command, control, and communications; mobility; and logistics. Each of these combat areas directly impacts upon the fire support analysis elements of fire mission generation, fire mission allocation, and fire support system effectiveness analysis. The quantification of this impact is a difficult undertaking, but necessary to assess the overall fire support system and identify trade-offs among these areas.

### C. Analysis Tools and Techniques

As indicated earlier in Figure 1, quantitative fire support analyses fall into three categories, namely, force level requirements analysis, operational concepts analysis, and technology assessment analysis. The decisionmakers generally associated with each of these categories of analyses as well as the general characteristics of the analyses are shown in Table 1.

Force level requirements analyses are required to support relatively "high level" decisionmakers, (e.g., strategists, force level planners, and multiforce commanders). These decisionmakers are interested in relatively aggregate measures of effectiveness and cost of fire support systems; therefore, multisided analysis using abstract models with aggregate scenario descriptions are employed. As a result, the types of analysis tools most applicable to these analyses are games (including war games, analytic games, and computerized games) and hybrids which incorporate both gaming and simulation techniques.

Operational concepts analyses are generally used in support of the operations level decisionmakers, including the tacticians, weapon system planners, and operational commanders. These decisionmakers are interested in fairly detailed analyses of the dependence of fire support system effectiveness and cost upon tactics, operational concepts, and weapon systems employment. The analyses techniques most applicable to this type of analyses include simulations, hybrids, and analytic models.

TABLE 1  
DECISIONMAKERS AND DISTINGUISHING CHARACTERISTICS ASSOCIATED WITH  
CATEGORIES OF FIRE SUPPORT ANALYSIS

FIRE SUPPORT ANALYSIS CATEGORY	DECISIONMAKERS USING THE ANALYSIS	DISTINGUISHING CHARACTERISTICS OF THE ANALYSIS
FORCE LEVEL REQUIREMENTS	<ul style="list-style-type: none"> <li>• STRATEGISTS</li> <li>• FORCE LEVEL PLANNERS</li> <li>• MULTIFORCE COMMANDERS</li> </ul>	<ul style="list-style-type: none"> <li>• ABSTRACT MODELS</li> <li>• AGGREGATE SCENARIO DESCRIPTIONS</li> <li>• MULTISIDED ANALYSIS</li> </ul>
OPERATIONAL CONCEPTS	<ul style="list-style-type: none"> <li>• TACTICIANS</li> <li>• WEAPON SYSTEM PLANNERS</li> <li>• OPERATIONAL COMMANDERS</li> </ul>	<ul style="list-style-type: none"> <li>• "REAL-WORLD" MODELS</li> <li>• DETAILED SCENARIO DESCRIPTIONS</li> <li>• ONE-SIDED ANALYSIS</li> </ul>
TECHNOLOGY ASSESSMENT	<ul style="list-style-type: none"> <li>• RESEARCH &amp; DEVELOPMENT PLANNERS</li> <li>• WEAPON SYSTEM DESIGNERS</li> <li>• SYSTEM TEST &amp; EVALUATION DIRECTORS</li> </ul>	<ul style="list-style-type: none"> <li>• VERY DETAILED HIGH-RESOLUTION SUBMODELS</li> <li>• LIMITED-SCOPE SCENARIO DESCRIPTIONS</li> <li>• ONE-SIDED ANALYSIS</li> </ul>

Technology assessment analyses provide technical decisionmakers such as research and development planners, weapon system designers, and system test and evaluation directors with information regarding alternative weapon technologies. The application of advanced technology to weapon systems and the cost-effectiveness of competing weapon designs are examples of the type of interests of this level of decisionmakers. Such information is provided only by very detailed submodels of the specific technical area of interest, usually with a one-sided scenario of limited scope. Simulations and analytic models are best suited for this type of analysis.

The analytic tools/techniques mentioned above are shown in Table 2 together with their distinguishing characteristics and the problem areas they address. It should be noted that all models do not neatly fall into one of these types of analysis tools. Sometimes a model incorporates some characteristics of several tools and the category it falls into will depend on the area emphasized by the user.

For completeness, it is useful to place such optimization techniques as linear programming, nonlinear programming, and dynamic programming in the context of quantitative fire support analyses. All of these techniques are concerned with allocating limited resources (weapon systems) among competing demands (missions) in an "optimal" manner. The major difficulty with applying these optimization techniques to fire support analysis is establishing the criterion which defines the "optimal" situation. Invariably the choices among alternative fire support systems depends upon qualitative considerations and numerous quantitative measures. Because the qualitative factors cannot generally be quantified and the relative influence of the numerous quantitative measures in the decisionmaking process is generally not quantifiable, these optimization techniques cannot be embedded into a general fire support analysis. However, in a specific analysis for a specific decisionmaker at a specific point in time, such an optimization technique may be valuable, but only if the decisionmaker is willing to precisely define, in quantitative terms, his decision process.

TABLE 2

## GENERAL DESCRIPTION OF FIRE SUPPORT ANALYSIS TOOLS

TYPE OF ANALYSIS TOOL	PROBLEM AREAS GENERALLY ADDRESSED	DISTINGUISHING CHARACTERISTICS
WAR GAMES	<ul style="list-style-type: none"> <li>● STRATEGIC</li> </ul>	<ul style="list-style-type: none"> <li>● TWO-SIDED ANALYSIS</li> <li>● ACCOUNTS FOR HUMAN FACTORS</li> <li>● VERY TIME-CONSUMING TO USE</li> </ul>
ANALYTIC GAMES	<ul style="list-style-type: none"> <li>● STRATEGIC</li> </ul>	<ul style="list-style-type: none"> <li>● TWO-SIDED ANALYSIS</li> <li>● PROVIDES "OPTIMAL" STRATEGIES</li> <li>● VERY ABSTRACT REPRESENTATIONS</li> <li>● PENCIL AND PAPER ANALYSIS</li> </ul>
COMPUTERIZED GAMES	<ul style="list-style-type: none"> <li>● STRATEGIC</li> </ul>	<ul style="list-style-type: none"> <li>● TWO-SIDED ANALYSIS</li> <li>● PROGRAMMED DECISION LOGIC</li> <li>● FAIRLY AGGREGATE ENGAGEMENT DESCRIPTIONS</li> <li>● FAIR AMOUNT OF COMPUTER TIME REQUIRED</li> </ul>
SIMULATIONS	<ul style="list-style-type: none"> <li>● TACTICAL</li> <li>● TECHNOLOGICAL</li> </ul>	<ul style="list-style-type: none"> <li>● ONE-SIDED ANALYSIS (GENERALLY)</li> <li>● CLOSE REPRESENTATION OF THE REAL-WORLD ENGAGEMENT DYNAMICS</li> <li>● DETAILED SCENARIO DESCRIPTIONS</li> <li>● SIGNIFICANT AMOUNT OF COMPUTER TIME REQUIRED</li> </ul>
ANALYTIC MODELS	<ul style="list-style-type: none"> <li>● TACTICAL</li> <li>● TECHNOLOGICAL</li> </ul>	<ul style="list-style-type: none"> <li>● ONE-SIDED ANALYSIS</li> <li>● FUNCTIONAL RELATIONSHIPS AMONG SYSTEM PARAMETERS</li> <li>● EASY AND INEXPENSIVE TO USE</li> </ul>
HYBRIDS	<ul style="list-style-type: none"> <li>● STRATEGIC</li> <li>● TACTICAL</li> </ul>	<ul style="list-style-type: none"> <li>● TWO-SIDED ANALYSIS (GENERALLY)</li> <li>● COMBINES GAMING, SIMULATION, AND ANALYTIC MODELING</li> <li>● SIGNIFICANT AMOUNT OF COMPUTER TIME REQUIRED</li> </ul>

#### IV FIRE SUPPORT DATA RESERVOIR

##### A. Role of the Data Reservoir

The third component of the structure for effective fire support analysis is the data reservoir. One role of the data reservoir is to provide the two-sided analyses data required to insure the validity of the many one-sided quantitative analyses that are performed in the area of fire support. For example in many analyses, the enemies' actions are assumed to be preprogrammed while the friendly forces' actions are varied to evaluate alternative force mixes, concepts of operations, weapons systems, etc. The data generated with two-sided analyses are essential to establish what the preprogrammed enemy actions should be and over what ranges of friendly force actions the preprogrammed actions remain valid. Thus, although it would be foolish, if not impossible, to attack all fire support problems with multisided analyses, data from multisided analyses must be utilized even for the one-sided analyses.

Another role of the data reservoir is to serve as a repository for the state-of-the-art knowledge of fire support analysis. Results from all fire support analyses are used to refine the data base and incorporate improved concepts of operations, force deployment, etc. Because the data base is a dynamic system it will require continuing management in updating data and providing inputs to analyses.

##### B. Types of Data

The type of information in the data base is generally aggregate measures which characterize the composition, concepts of operation, and engagement features of both friendly and enemy major organizational units. For example, such measures might include statistical distribution of the types of individual components of each organizational unit, movement rates, attrition rates, and firepower coefficients. It is essential to include aggregate rather than detailed parameters to keep the data

storage requirements within reason and to insure that the data can be catalogued and organized in a usable form.

An important task in the development of a fire support data base is to isolate a reasonable set of measures which adequately characterizes the forces, operational concepts, engagement features, etc., that are of concern in fire support analysis. These measures are referred to as planning factors. It is necessary that each of these factors be quantifiable and measurable. It is also necessary that from this set of aggregate measures, more detailed characterizations are derivable. For example, a planning factor may be the mean and variance of the numbers of tanks, trucks, and armored personnel carriers in a Soviet motorized division. A target generator which would generate the actual number of each type of vehicle from these statistics could easily be devised if needed for a specific analysis.

#### C. Source of Data

One source of data for the data base is empirical information. This includes actual battlefield statistics, intelligence information, and judgments by experts. Since wars are not fought to gather empirical data for the analyst, very little battlefield data is available. Of the data that was gathered, much of it is unusable because of its incompleteness, form, or dubious validity. Therefore, most of the data for the data base must be derived from analytical investigations.

A major analytic technique for generating data for the data base is the use of two (or more) sided engagement analyses which have been referred to as games in this work. Traditional war games (both manual and computer augmented), analytic games, and computerized games all belong to the general category of games. The essential feature of games is that they are multisided, that is, they involve more than one party competing to achieve conflicting objectives.

Another source of data is the results of the various force level requirements, operational concepts, and technology assessment studies which are fed back into the data base after the data from these studies

are transformed into the appropriate planning factors. The results of these studies are thus used to continually refine the data base. This leads to more accurate data for deriving inputs to fire support analyses.



## V CONCLUSIONS

In developing the analysis framework, several fundamental conclusions became evident. The first is that the purpose of fire support analysis is to support, not supplant, the human decisionmakers who must integrate the results of quantitative analysis with qualitative factors and constraints (economic, political, etc.). With the diversity and fluidity of the qualitative concerns in fire support decisions, any attempt to replace the human decisionmakers via some analytical or optimization technique is doomed to failure. Thus, the objective of quantitative analysis is to provide the decisionmaker with support in the form of decision aids which depict the tradeoffs associated with key system parameters and identify sets of alternative systems which appear promising in view of the quantitative factors considered in the analysis.

A second fundamental conclusion is that no single fire support analysis "model" can be developed that will fulfill all fire support analysis requirements. The particular types of problems that are of concern to decisionmakers at different levels vary widely. As a result, the analytic tools and techniques applicable for providing quantitative support differ among decisionmakers, not only among the various levels, but even among decisionmakers at the same level. The aim should be to incorporate existing fire support analysis models (after modifying them to conform to a set of standards) into a fire support analysis package. Additional models would be developed and incorporated into this package as needed to supplement the existing package. Access to this standard fire support analysis package would provide analysts with a common base for providing quantitative support for fire support decisionmakers. Procedures for standardization, assembly, and management of such an analysis package should be given considerable attention.

A third conclusion is that a data reservoir which provides the necessary information for developing inputs to fire support analysis and for pooling the results of past and future fire support analyses is needed. Even though different fire support analyses require scenario input data in different forms and to different degrees of detail, this data base will provide the means to insure consistent scenario descriptions. In addition, by incorporating the results of all fire support analyses, this data reservoir will provide a repository for the state-of-the-art knowledge of fire support.

Finally, considerable efforts must be expended in the development of the fire support analysis structure before any additional model building is undertaken. Work is required in the area of defining a hierarchy of fire support decisionmakers and the associated hierarchy of measures of effectiveness, which will include working closely with the decisionmakers. Work is also required to design and synthesize the required data reservoir. This will include defining appropriate planning factors, data structure, and means of maintaining and managing this data reservoir.

Work must also be undertaken to structure the fire support analysis package. Standards (e.g., regarding inputs, outputs, etc.) will have to be established to insure the integrity of the package and the compatibility of the various models in the package. Having structured the package, presently developed fire support analysis models will have to be incorporated into the package. Only then can gaps in the analysis package, which may be filled by additional model building, be identified.

#### REFERENCES

1. "Department of Defense Dictionary of Military and Associated Terms," JCS Pub 1, The Joint Chiefs of Staff, Washington, D.C. (1 January 1972).
2. "General Operational Requirement (GOR), SPA-1, Supporting Arms," Department of the Navy, Headquarters U.S. Marine Corps, Washington, D.C., (April 1972) SECRET/NOFORN.
3. G. D. Brewer, "An Analyst's View of the Uses and Abuses of Modeling for Decisionmaking," The RAND Corporation, P-5395, (January 1975).
4. J. R. King, "Probability Charts for Decisionmaking," Industrial Press, Inc., (1971).
5. R. A. Howard, et al, editors, "Readings in Decision Analysis," Decision Analysis Group, Stanford Research Institute, Menlo Park, California (1974).
6. R. J. McNicholas and F. L. Crane, "Guide to Fire Support Mix Evaluation Techniques," Vol 1, Contract N00014-71-G-0417, Naval Warfare Research Center, Stanford Research Institute, Menlo Park, California (March 1973).

APPENDIX 3

SOLUTION OF A LARGE SCALE AIRFORCE

ORDNANCE PLANNING PROBLEM BY

MATHEMATICAL PROGRAMMING

SOLUTION OF A LARGE SCALE AIRFORCE  
ORDNANCE PLANNING PROBLEM BY  
MATHEMATICAL PROGRAMMING

George B. Dantzig,<sup>1</sup> John Friel,<sup>2</sup> Robert Golightly,<sup>2</sup>  
Roy P. Harvey,<sup>3</sup> Robert D. McKnight<sup>3</sup>

ABSTRACT

This paper summarizes a larger paper<sup>4</sup> dealing with the solution of a non-linear mathematical programming model which is approximated by a sequence of linear programs with about 400 constraints each. The major point of interest is that each of the problems may have up to several million variables so that much of the project effort has been to develop an efficient pricing algorithm and strategy.

The all FORTRAN software system, named TAC RESOURCER, uses a flexible mathematical programming code, CAMPS, which has been adapted to take advantage of problem structure and characteristics. In this case the data storage task is kept within reason by a column generation scheme and knowledge of the problem characteristics makes special sensitivity analyses possible.

---

<sup>1</sup>Control Analysis Corporation and Stanford University.

<sup>2</sup>USAF, Studies and Analysis, Washington, D. C.

<sup>3</sup>Control Analysis Corporation.

<sup>4</sup>George B. Dantzig, John Friel, Robert Golightly, Roy P. Harvey and Robert D. McKnight, Solution of A Large-Scale Mathematical Programming Problem Related to Ordnance Planning, Control Analysis Corporation, Palo Alto, California, September 1975.

## TABLE OF CONTENTS

SECTION	PAGE
1	General Description of Problem and Approach . . . . . 3
2	Formulation as a Non-Linear Mathematical Program . . . . 5
3	The Linear Program Formulation for one Time Period . . . 11
4	The Computer System . . . . . 14
5	The Column Generation Algorithms . . . . . 17
6	Sensitivity Features . . . . . 22
7	Results and Conclusions . . . . . 24

## 1. General Description of Problem and Approach

This paper describes a large scale ordnance planning and resource allocation model which has been developed for the U. S. Air Force. The model is designed to aid in the following three analyses:

- 1) Determination of alternative stockpiles of air-to-ground munitions.
- 2) Determination of the most effective weapon modules for the modular weapon development program.
- 3) Allocation of air-to-ground resources between aircraft and weapons.

The planning horizon for the model consists of several time periods. A planning horizon, for example, might be 180 days broken down into four time periods of 10, 30, 50 and 90 days. A feature which makes the problem of special interest is that in general, it contains a very large number of variables, (several million perhaps), representing sortie types with all permissible combinations of aircraft type, ordnance type, target type, delivery condition, weather state and time period.

In Section 2 the problem is formulated as a convex non-linear program. At the present time the model is solved suboptimally as a sequence of linear programs, one for each time period. The formulation of the linear program for one time period is described in Section 3.

The model is handled completely by a computer system named TAC RESOURCER within which is embedded a flexible linear programming code CAMPS. The non-linear functions are represented by piecewise linear approximations within TAC RESOURCER. Because of the very large number of variables available

for the mathematical program for each time period, we do not follow the usual approach taken in large mathematical programming computer systems of constructing a packed word matrix consisting of all the nonzero coefficients of the constraints and objective rows with row and column identifications. The basic information necessary for constructing these nonzero elements is provided to the system in the form of a large, sequentially organized data base produced by a different suite of computer programs. This data base changes relatively infrequently. As a preliminary to solving the mathematical program the data base is processed and the required information is extracted from it and stored concisely in so called "packages" of information in random access file organization. Subsequently, during simplexing, these "packages" are brought into high speed core according to certain rules and opened up for pricing. The computer system and the algorithms employed in the packaging and pricing procedures are described in Sections 4 and 5 and the post optimal sensitivity analyses are described in Section 6. Section 7 contains some results and conclusions.



## 2. Formulation as a Non-Linear Mathematical Program

Let

$i$  = the index of aircraft type

$j$  = the index of ordnance type

$k$  = the index of target type

$d$  = the index of delivery condition

$l$  = the index of weather state

$t$  = the index of time period

$m$  = the index of ordnance class containing ordnance type

indices  $J_m$ .

Let the variable  $X_{ijkdlt} \geq 0$  be the number of sorties flown by the  $i^{\text{th}}$  aircraft type with the  $j^{\text{th}}$  ordnance type against the  $k^{\text{th}}$  target type with the  $d^{\text{th}}$  delivery condition in the  $l^{\text{th}}$  weather state, in the  $t^{\text{th}}$  time period. The notation  $X_{ijkdlt}$  refers only to permissible sortie types. Combinations of indices which do not give rise to permissible sorties are omitted so that, for example  $\sum_{jk} X_{ijkdlt}$  means that the summation is carried out only for permissible  $jk$  combinations for the given  $idlt$  combination. Multisubscripted variables (or constants) that are summed over all values of a subscript, will be so indicated by replacing the subscript by a dot. Thus

$$X_{i \dots dlt} = \sum_{jk} X_{ijkdlt}$$

In certain cases weighted combination of the X's will be similarly treated, e.g.

$$E \dots k \dots t = \sum_{ijdl} E_{ijkd} X_{ijkdlt}$$

The following notations refer to known constants:

- $V_{kt}$  = value of a target of type  $k$  in time period  $t$ .
- $T_{kl}$  = total number of  $k$  type targets at beginning of time horizon.
- $E_{ijkd}$  = average number of targets killed on a single sortie with indices  $ijkdlt$  assuming that at least  $E_{ijkd}$  'live' targets of type  $k$  remain at risk and that there is no diminishing sortie effectiveness due to difficulty in distinguishing live targets from 'dead' ones.
- $\hat{A}_{ijkd}$  = average attrition of a sortie with indices  $ijkdlt$ .
- $W_{ijk}$  = ordnance load per sortie of type  $ijkdlt$ .
- $\mu_{1kt}$  = a factor referring to targets killed in previous time period and not available for current time period  $t$ .
- $\mu_{2kt}$  = a factor referring to targets killed two periods earlier and now reconstituted and available for attack in period  $t$ .
- $\left. \begin{matrix} B_{lit} \\ B_{2jt} \\ B_{mt} \\ B_t \end{matrix} \right\}$  = budgetary bounds on aircraft types, ordnance types, ordnance classes and overall for time period  $t$ .

### Objectives

$$\max \sum_t \sum_k v_{kt} T_{kt} \left[ 1 - \exp(-E_{k,t} / T_{kt}) \right] \quad (1)$$

where  $T_{kt}$  is the number of undestroyed type  $k$  targets remaining in period  $t$  defined as:

$$T_{k2} = T_{k1} - \mu_{1k2} M_{k1} \quad (2)$$

$$T_{kt} = T_{k,t-1} - \mu_{1kt} M_{k,t-1} + \mu_{2kt} M_{k,t-2}, \quad t > 2 \quad (3)$$

where  $M_{kt}$  refers to the number of type  $k$  targets killed in time period  $t$ .

The objective function is one used by the Air Force for ordnance planning and represents the total expected damage in terms of military worth. As is shown in the larger paper, [4] it is a concave function in its arguments and can be derived by applying a binomial attrition process and making such assumptions as:

Every aircraft type being considered is capable of reaching all targets of each given type with enough fuel to expend its ordnance and return. Each sortie is flown independently of all others.

On each sortie the aircraft locates and attacks a target or targets of only one type.

Each sortie expends its entire load of ordnance, using one of a pre-selected set of delivery tactics.

Implicit in this objective function is the increasing difficulty of finding a 'live' target to attack. As live targets are neutralized, the effectiveness of subsequent sorties decreases. This decreasing effectiveness stems from the assumption that expenditures of ordnance occur against targets which may have been killed by earlier sorties.

#### Sortie vs. Attrition

The ability to fly sorties is affected by the attrition encountered. If attritions are heavy, the upper limit on the number of sorties is lower than if attritions are light. This relationship may be derived as follows:

Let  $N_{it}$  be the number of available aircraft of type  $i$  at the beginning of time period  $t$ ,  $P_{it}$  be the sortie potential per aircraft, assuming no attritions occur, and  $\hat{A}_{it}$  the expected loss of type  $i$  aircraft during period  $t$ . Then for any one aircraft of type  $i$  the probability of attrition during the time period is  $\hat{A}_{it}/N_{it}$  and the corresponding probability of survival is  $1 - \hat{A}_{it}/N_{it}$ . Assuming a uniform attrition rate over the time period, it follows that the probability of survival of one aircraft from one sortie is  $(1 - \hat{A}_{it}/N_{it})^{1/P_{it}}$  on average. Using the multiplication principle for probabilities and assuming that  $P_{it}$  is a positive integer, the expected maximum number of sorties for one aircraft becomes:

$$\sum_{k=0}^{P_{it}-1} \left\{ (1 - \hat{A}_{it}/N_{it})^{1/P_{it}} \right\}^k = \frac{\hat{A}_{it}/N_{it}}{1 - (1 - \hat{A}_{it}/N_{it})^{1/P_{it}}} \quad (4)$$

and for  $N_{it}$  aircraft it becomes:

$$U_{it}(\hat{A}_{it}) = \frac{\hat{A}_{it}}{1 - (1 - \hat{A}_{it}/N_{it})^{1/P_{it}}} \quad (5)$$

In the larger paper, [4] a general proof of the concavity of  $U_{it}$  is given for real  $P_{it} \geq 1$ . For the case that  $P_{it}$  is a positive integer, (4) shows that  $U_{it}$  is the sum of concave functions of the arguments  $A_{it}$  and is therefore concave.

The above derivation allows us to impose the constraint:

$$x_{i \dots t} \leq U_{it}(\hat{A}_{it}), \text{ where } \hat{A}_{it} = A_{i \dots t}, \quad (6)$$

However for practical reasons the constraint actually imposed is the equivalent inverse relation:

$$\hat{A}_{i \dots t} \leq U^{-1}(x_{it}), \text{ where } x_{it} = x_{i \dots t} \quad (7)$$

#### Other Non-linearities

Non-linear budgetary constraints in each time period may be imposed on several different linear forms in the problem. For example:

$$\bar{C}_{1it}(x_i \dots t) \leq B_{1it} \quad (8)$$

$$\bar{C}_{2jt}\left(\sum_{\tau=1}^t W_{\tau j} \dots \tau\right) \leq B_{2jt} \quad (9)$$

where  $\bar{C}_{1it}$  and  $\bar{C}_{2jt}$  above are functions of the arguments shown in parenthesis. The set  $\{Z|Z \geq 0, \bar{C}_{pqt}(Z) \leq B_{pqt}\}$  is convex for  $pq=1i$  or  $2j$ . Constraints (8) and (9) are readily transformed into the equivalent linear bounds.

$$x_i \dots t \leq \bar{C}^{-1}(B_{1it}) \quad (10)$$

$$\sum_{\tau=1}^t W_{\tau j} \dots \tau \leq \bar{C}_{2jt}^{-1}(B_{2jt}) \quad (11)$$

Other budgetary constraints involve convex functions  $C_{1i}$  and  $C_{2j}$  and are of the form:

$$\sum_{j \in J_m} C_{2j}\left(\sum_{\tau=1}^t W_{\tau j} \dots \tau\right) \leq B_{mt} \quad (12)$$

$$\sum_i C_{1i}\left(\sum_{\tau=1}^t x_i \dots \tau\right) + \sum_j C_{2j}\left(\sum_{\tau=1}^t W_{\tau j} \dots \tau\right) \leq B_t \quad (13)$$

### Other Constraints

The other constraints in the problem are linear and consist of lower and/or upper bounds on various weighted combinations of the variables. They are not included in this section; but Section 3 will list them (in a consolidated form) as they appear in the linear programs.

### 3. The Linear Program Formulation for One Time Period

In this section we describe the linear program which approximates one time period of the non-linear program described in Section 2. We continue to use the same notation and introduce more as required. The subscript  $t$  is dropped.

We will introduce non-negative variables  $y_{1jr}$ ,  $y_{2jr}$ ,  $y_{3kr}$  bounded above by 1.0 that are used to achieve a polygonal approximation to the non-linearities. These approximations are constructed automatically by TAC RESOURCER according to maximum error tolerances provided by the user. However, we will not describe these methods in this short paper. Instead, we assume that a satisfactory approximation is achieved by sets of coefficients  $(a_{pqr}, b_{pqr}, d_{pqr})$  where  $a_{pqr}$  measure changes in arguments, and  $b_{pqr}$  and  $d_{pqr}$  measure the corresponding changes in the functions.

### Objective

Maximize  $W$ , where

$$W - \sum_{kr} b_{3kr} y_{3kr} = 0 \quad (14)$$

where the arguments are defined by the constraints:

$$\sum_r a_{3kr} y_{3kr} - E_{..k..} = -\alpha_{3k} \quad (15)$$

and  $\alpha_{3k}$  and  $\alpha_{3k} + \sum_r a_{3kr}$  account for bounds on  $E_{..k..}$

#### Sortie vs. Attrition

$$- \sum_r d_{1ir} y_{1ir} + \hat{A}_i \dots \leq 0 \quad (16)$$

with the arguments defined by:

$$- \sum_r a_{1ir} y_{1ir} + X_i \dots = \alpha_{1i} \quad (17)$$

where  $\alpha_{1i}$  and  $\alpha_{1i} + \sum_r a_{1ir}$  establish bounds on  $X_i \dots$

#### Budgetary Constraints

The same  $y_{1ir}$  variables and definitional constraints (17) are used for the sortie cost in the budgetary constraints. Thus they become:

$$\sum_{j \in J_m, r} b_{2jr} y_{2jr} \leq B_m - \alpha_{4m} \quad (18)$$



$$\sum_{ir} b_{1ir} y_{1ir} + \sum_{jr} b_{2jr} y_{2jr} \leq B - \alpha_0 \quad (19)$$

where the ordnance arguments are given by the constraint:

$$- \sum_r a_{2jr} y_{2jr} + W_{.j...} = \alpha_{2j} \quad (20)$$

and  $\alpha_{4m}$ ,  $\alpha_0$ ,  $\alpha_{2j}$  account for lower bounds.

#### Other Constraints

In order to describe the constraints not implicit in the bounds imposed by the  $\alpha$ 's and  $\sum_r a$ 's we shall use the symbol  $\overset{*}{=}$  to represent one of the symbols  $\leq$ ,  $=$  or  $\geq$ . The letters  $R_{il}$ ,  $T_{kl}$  and  $G_{ik}$  are known constants.

$$X_{i...l} \overset{*}{=} R_{il} \quad (21)$$

$$X_{i.k...} \overset{*}{=} G_{ik} \quad (22)$$

$$E_{...k.l} \overset{*}{=} T_{kl} \quad (23)$$

#### 4. The Computer System

The overall flow diagram of the system is illustrated in Fig. 1. The program is coded almost entirely in FORTRAN and currently consists of about 14,000 statements used in an overlay system which uses about 630k bytes of core storage.

There are two main sources of input; the data base which is maintained by an independent system, (see Section 5) and the so-called parameter input data which determines the structure of the problem in terms of the subset of variables, the constraints employed, the non-linear functions used, etc. The latter also contains parameters that control the LP solution algorithm and select the post optimal analyses.

Much of the system relates to editing and processing input data, forming the problem at each stage and producing reports of the solutions which are both cumulative and non cumulative over time. It is possible to carry out sensitivity ranging analyses on parameters of the system singly or in some cases, collectively (see Section 6). The system is heavily loaded with user parameters and options, which, if not specified explicitly, will take on default values. Within a time period data specification is order independent. If desired the system will operate in an initial mode which processes and edits all data for all time periods without, however, solving any of the linear programs. Only if everything is apparently in order does the system go into its second mode and commence to solve the linear programs.

SYSTEM FLOW DIAGRAM - TAC RESOURCER

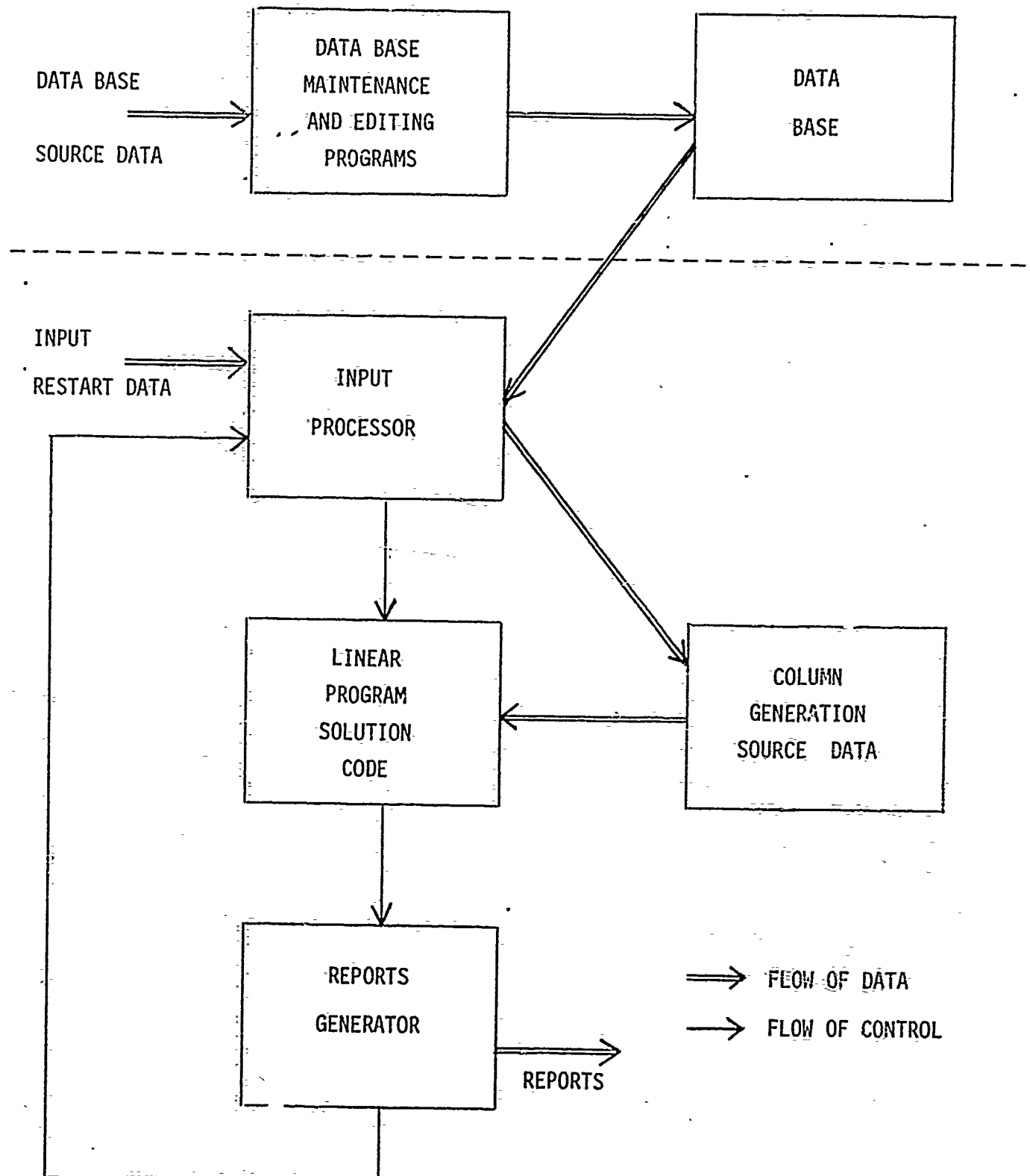


Figure 1

Across time periods the structure of the linear programs may change. For example, a new aircraft type may become available in a later time period, or certain combinations of aircraft type and ordnance type may be available in certain time periods and not in others.

Getoff and restart capabilities are available which allows the user to terminate a run with all pertinent arrays saved after a specified time, number of simplex iterations or a number of time periods have been dealt with. The data for succeeding time periods may be changed if desired, and the program may be restarted from point of exit using a special restart parameter card. In case of machine or operator malfunction or other unscheduled termination, the program may always be restarted from the last time period processed.

The linear programming code is a modularized set of Fortran subroutines which is easily adapted to take advantage of special features of a problem. It has upper bounding and uses the product form of the inverse with double precision arithmetic where pertinent.

## 5. The Column Generation Algorithms

There are three types of linear programming columns in this problem. The slack variables and the upper bounded  $y$  variables associated with the piecewise linear approximations are kept in high speed core while a linear program is being solved.

From the linear programming standpoint the major point of interest is the handling of the enormous number of  $X_{ijkdlt}$  variables in a problem.

Each  $X$  variable has at most seven nonzero coefficients in its column arising from equations of type (15), (16), (17), (20), (21), (22) and (23).

The formula for computing the reduced cost of such a column is given by

$$\begin{aligned} \delta_{ijkdlt} = & \pi_{21lt} + E_{ijkd}\pi_{23kt} + \pi_{22kt} + \pi_{17lt} \\ & + W_{ijk}\pi_{20jt} - E_{ijkd}\pi_{15kt} + \hat{A}_{ijkd}\pi_{16lt} \end{aligned} \quad (24)$$

where the  $\pi$ 's are the simplex multipliers, using a notation which corresponds to the row numbers.

It can be seen that a matrix column can be constructed if three coefficients are known and that these coefficients are shared by many other columns. The three coefficients  $E_{ijkd}$ ,  $W_{ijk}$  and  $\hat{A}_{ijkd}$  are stored in a compact form in 'packages' in the random access storage. This packaging is

carried out after the structure of the linear program is known in a preliminary processing operation at the beginning of the problem run, and again for subsequent time periods if problem structure changes require it. The data base which contains these coefficients is processed and all required data extracted using an algorithm which attempts to maximize the amount of useful information in each record. Solid blocks of data in the data base are searched for. Gaps in the coefficient arrays indicate non-permissible sortie combinations.

After the packages are formed and depending on the availability of core storage, two parameters  $K_1$  and  $K_2$  are computed. The first parameter  $K_1$  is the minimum number of representative columns which will be held in core from each package of columns. The second parameter  $K_2 > K_1$  is the number of columns which is sought from a package each time it is brought in from random access storage. The main idea in the pricing strategy is to maintain in core representative columns from each of the packages as illustrated in Fig. 2.

A major iteration in the solution procedure is roughly as follows:

- 1). Select a package for pricing. Give priority to packages which do not have the requisite number of representatives. Otherwise determine which variable  $X_{ijkd/t}$  in core has the minimum reduced cost. Select the package which contains this variable. Initiate a read for the chosen package.
- 2). Simplex on the in core matrix until the most attractive reduced

# PACKAGE SELECTION

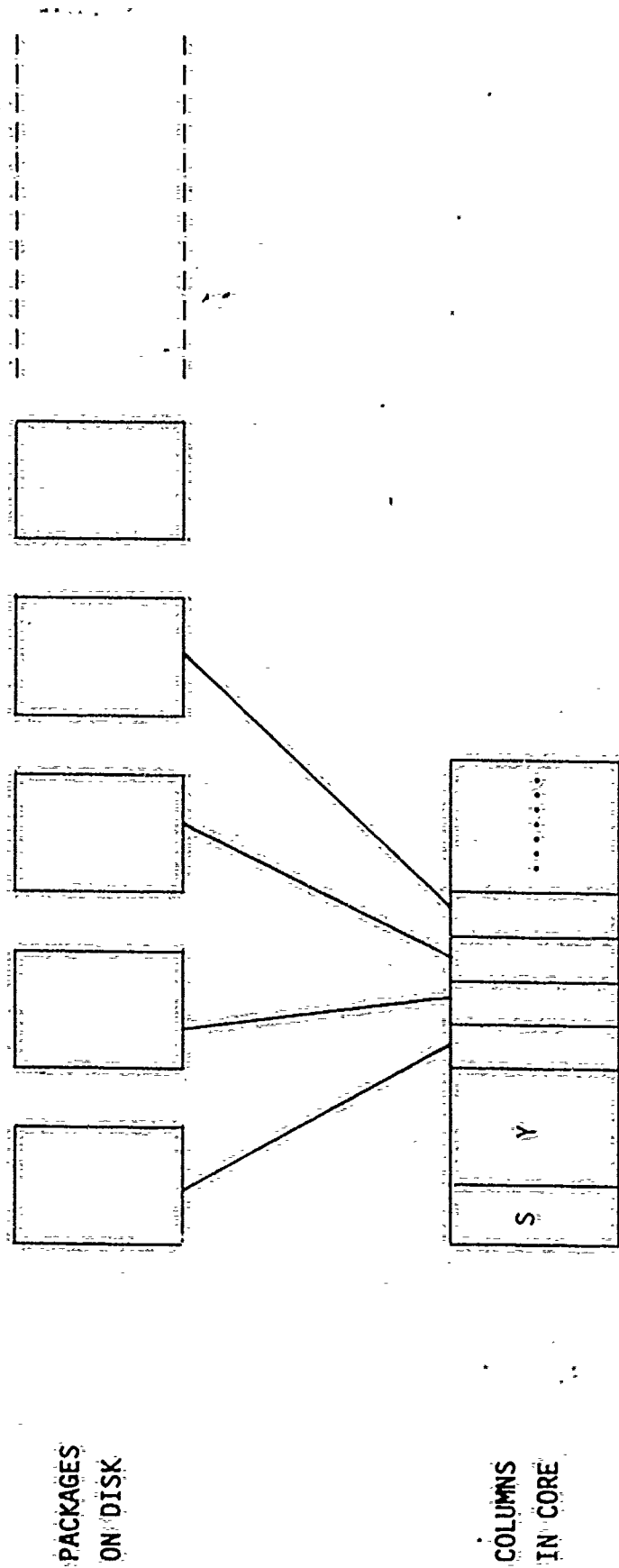


Figure 2

cost is greater than a tolerance  $\epsilon$ . If any simplex iterations occur, unlabel any labeled packages (see step 4).

- 3). According to a set of priority rules earmark  $K_2$  column spaces in core which may be used to record the chosen columns from the next package pricing operation. This may be 'new' core storage, or space occupied by 'stale' package representative columns.
- 4). Price out the 'package' in core. Choose up to  $K_2$  columns according to attractiveness. If no column has a negative reduced cost, label this package. If all packages are labeled, exit.
- 5). Return to step 1.

The tolerance  $\epsilon$  is a dynamic tolerance. It starts with a substantially negative value and gets increased when certain conditions occur, such as when a certain proportion of packages get labeled or very little progress is made at step 2). Its function is to attempt to strike a balance between computation work and real progress.

There is a complication in the above procedure in that one package is being processed while another is being transmitted from peripheral device to core. The simplex multipliers used in step 4) are therefore not current.

The pricing out of a package in step 4) is carried out in a 'nested' fashion according to the following table showing five major loops:



<u>Loop</u>	<u>Fetch</u>	<u>Compute</u>
i	$\pi_{17it}, \pi_{16it}$	
j	$\pi_{20jt}$	
k	$w_{ijk}, \pi_{15kt}, \pi_{22ikt}$	$\delta_{ijkt} = \pi_{17it} + w_{ijk}\pi_{20jt} + \pi_{22ikt}$
d	$e_{ijkd}, \hat{A}_{ijkd}$	$\delta_{ijkdt} = \delta_{ijkt} - e_{ijkd}\pi_{15kt}$ $+ \hat{A}_{ijkd}\pi_{16it}$
l	$\pi_{21ilt}, \pi_{23klt}$	$\delta_{ijkdlt} = \delta_{ijkdt} + \pi_{21ilt}$ $+ e_{ijkd}\pi_{23klt}$

The quantities  $\delta_{ijkt}$  and  $\delta_{ijkdt}$  are partial summations used in the nesting procedure.

## 6. Sensitivity Features

TAC RESOURCER allows the sensitivity of nearly all of the problem parameters to be studied in some manner. The primary method is ranging. Ranging is the technique that determines the range of values that a parameter (or set of parameters) may take while still maintaining the feasibility and optimality of the basis. This technique displays the revised solution values (primal and/or dual) at the extremes of the range and indicates the nature of these range limits.

Closely related techniques produce lists of non-basic variables that (most nearly) price at zero. The lengths of these lists can be determined by the user and are categorized in several ways. Another sensitivity report interprets the dual variables in management terms and relates these variables to the appropriate problem parameters.

Below we will name the algorithms available. No attempt will be made to explain these algorithms in this report.

- 1). Right-hand-side ranging with upper bounded variables.
- 2). Right-hand-side ranging with upper bounded variables, and with special designed techniques to account for implicit appearance of the parameter in the problem in places other than the right-hand-side.
- 3). Cost-row ranging.

These first three algorithms can range specific parameters individually or range a specified set of parameters simultaneously. In the latter case the user provides a vector of rates of change relative to the rate of change in an independent change parameter.

- 4). Row ranging.
- 5). Row ranging with special non-linear treatment to handle the sortie vs attrition relation.
- 6). Non-basic element cluster ranging.
- 7). Lists of (near) alternate basic variables. These lists may be:
  - (a) The non-basic variables with smallest 'reduced cost'.
  - (b) For each  $k$ , the non basic variables requiring the least increase in  $E_{ijkd}$  to have reduced cost of zero.
  - (c) For each  $i$ , the non-basic variables requiring the least decrease in  $\hat{A}_{ijkd}$  to have reduced cost of zero.

## 7. Results and Conclusions

TAC RESOURCER has been operational for more than a year during which time it has been used to solve more than 100 problems. None of the problems solved so far have been as large as originally contemplated. Typically a problem might contain 30 aircraft types, 50 ordnance types, 50 target types, 6 delivery conditions, 6 weather states and 10 time periods. A systematic study of solution strategies has not yet been undertaken.

A problem with about 250 rows and 100,000 variables per time period, with 10 time periods takes about 75 minutes cpu time to solve on an IBM 360/75 computer.

Acknowledgements

This research was sponsored by Hq USAF, Assistant Chief of Staff, Studies and Analysis, Washington, D. C. 20330, under contract number F44620-73-C-0079. The United States Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation hereon.

APPENDIX 4  
THE ROLE OF DIFFERENTIAL MODELS OF COMBAT  
IN FIRE SUPPORT ANALYSES

## THE ROLE OF DIFFERENTIAL MODELS OF COMBAT IN FIRE SUPPORT ANALYSES

by W. P. Cherry

### 1.0 INTRODUCTION

This paper addresses the use of one analytic methodology, namely differential models of combat, to describe fire support activities. The contents of the paper are based on the experience of Vector Research, Incorporated (VRI), in developing and using differential models in numerous military studies including studies of specific fire support requirements and roles. The remainder of the paper is organized as follows. Section 2.0 consists of a brief discussion of forerunner of the differential models, namely the Lanchester theory of combat, and the battalion level differential combat models developed by VRI. Section 3.0 is devoted to a discussion of VRI's current approach to fire support processes in combat models and addresses the critical elements of the processes and the means chosen to represent these elements. Section 4.0 is a description of VECTOR-2, a theater level combined arms model under development at VRI. The description is included to illustrate the degree to which fire support is modeled in a large scale model and to serve as a possible basis upon which future analytic needs can be defined.

## 2.0 BACKGROUND

In their simplest forms the equations proposed by Lanchester (1916) to represent combat are as follows:

$$\frac{dx}{dt} = -ay$$

$$\frac{dy}{dt} = -bx$$

and

$$\frac{dx}{dt} = -axy$$

$$\frac{dy}{dt} = -bxy$$

where

$x$  = the number of Blue survivors at time  $t$ ,

$y$  = the number of Red survivors at time  $t$ ,

$a$  = the rate at which a single Red element attrits Blue elements, and

$b$  = the rate at which a single Blue element attrits Red elements.

The first two equations are usually described as representing the "square law" and are appropriate for aimed fire and negligible target acquisition times, while the third and fourth equations are described as representing the "linear law" and are appropriate for area fire in which target acquisition times are relatively larger than times to destroy acquired targets and are inversely proportional to target density. (For a more complete discussion of the Lanchester models see Weiss [1957]).



Although the simplicity of the Lanchester model is attractive, it can rarely be applied to produce realistic results for analyses of questions involving modern weapons systems, force structures and tactics. In particular the attrition rates,  $a$  and  $b$  above, which were assumed constant by Lanchester are in fact variable and extremely complex functions of the state of the battle at any given time. Moreover, studies of the dynamics of combat have indicated that the categorization of lethality mechanisms into either linear or square forms is incomplete, since there are mechanisms in which, for example, the attrition produced by a single weapon system is not proportional to the number of targets but depends upon this number in a complicated way. As a consequence, a class of combat models has been developed which most properly might be designated *differential* models of which the Lanchester models are a special case. VRI has been and is currently engaged in research on differential models of combat; this research and some of its results are described below.

### *2.1 The Differential Methodology*

In a broad sense the primary objective of our research is in the development of analytic structures that can be used to predict the history of an engagement. Essentially, this would be a trajectory or trace of time, geometry, casualties, and resources expended for both forces. Measures of combat effectiveness such as the ratio of surviving forces at the objective, time to overrun the objective, and the amount of terrain controlled are then determined from these results of battle.

Ideally, there exists some functional relationship between the results of battle and the initial numbers of forces, types and capabilities of the weapons systems, the doctrine of employment, and the environment. Thus, we would like to specify the function  $f$  shown below.

$$\begin{pmatrix} \text{Results} \\ \text{of} \\ \text{Battle} \end{pmatrix} = f \begin{cases} \text{Numbers of Forces} \\ \text{Types of Weapon Systems} \\ \text{Weapon Capabilities} \\ \text{Doctrine of Employment} \\ \quad \text{(tactics, organization)} \\ \text{Environment} \end{cases}$$

Unfortunately, it is not known how to construct such a function directly, nor is there sufficient data to develop it empirically. Because of this, we attempt to approximate what happens in a small period of time during the battle. That is, for each side, it is hypothesized that in a short period of time

- (1) locations change due to tactical movement,
- (2) weapon systems are attrited by enemy activity,
- (3) resources are expended, and
- (4) personnel become casualties due to enemy activity.<sup>1</sup>

Focusing on the loss of weapon systems and personnel, it is assumed that, if the state of the battle at the beginning of the small interval is known, and the activity that takes place during the interval is known, the *rate* at which weapons systems and personnel are attrited during this small interval can be predicted. It is because of this rate focus that the mathematical structure employed to model the combat activity is that of differential equations.

---

<sup>1</sup>Reserve commitment and resupply during the small interval of time are also possible but are omitted for presentation purposes.

Mathematically, these assumptions take the form of the following coupled sets of differential equations:<sup>1,2</sup>

$$\frac{dn_j}{dt} = \sum_{i=1}^I A_{ij}(r)m_i \quad \text{for } j = 1, 2, \dots, J$$

$$\frac{dm_i}{dt} = - \sum_{j=1}^J B_{ji}(r)n_j \quad \text{for } i = 1, 2, \dots, I$$

where

$A_{ij}(r)$  = the utilized per system effectiveness in the  $i^{\text{th}}$  Blue group against the  $j^{\text{th}}$  Red target at range  $r$ . This is called the Blue attrition coefficient.

$B_{ji}(r)$  = the utilized per system effectiveness of systems in the  $j^{\text{th}}$  Red group against the  $i^{\text{th}}$  Blue target at range  $r$ .

This is called the Red attrition coefficient.

It is noted that this formulation is a deterministic one which treats the numbers of surviving forces ( $m_i$  and  $n_j$ ) as continuous variables, while clearly the actual battle activity is a random phenomenon and  $m_i$  and  $n_j$  are integer-valued variables. Although many probabilistic arguments are contained in this formulation, the output of the model is a deterministic trajectory of the surviving numbers of forces. It is of

<sup>1</sup>Although the variable  $r$  is used to designate the range between the firing weapon group and the target group, it should be noted that, in application of the model, actual time trajectories and positions of each group can be considered.

<sup>2</sup>Although not explicitly shown, resources expended are explicitly contained in the development of the  $A_{ij}$  and can be determined directly.

interest to note that research done on comparing deterministic and stochastic formulations for the homogeneous-force case (only one force group on each side) indicates that the deterministic formulations are reasonably good approximations of the expected number of survivors if there is a small probability that either side is annihilated. Additionally, in many defense studies that employ Monte Carlo simulations, typically only the expected results are considered in the decision-making process.

In the computational program, these differential equations are approximated by the difference equations.

$$m_i(t + \Delta t) = \max \left\{ 0, m_i(t) - \sum_{j=1}^J B_{ji}(t) n_j(t) \Delta t \right\}$$

for  $i = 1, 2, \dots, I$

$$n_j(t + \Delta t) = \max \left\{ 0, n_j(t) - \sum_{i=1}^I A_{ij}(t) m_i(t) \Delta t \right\}$$

for  $j = 1, 2, \dots, J$ .

where  $\Delta t$  is the computational time step. The correspondence between battle time and spatial distribution of forces during the battle is obtained from knowledge of the movement pattern of all Red and Blue groups.

The attrition coefficients ( $A_{ij}$  and  $B_{ji}$ ) are, as one would expect, complex functions of the weapon capabilities, target characteristics, distribution of the targets, allocation procedures for assigning weapons to targets, etc. The model attempts to reflect these complexities by partitioning the total attrition process into four distinct ones:

- (1) the effectiveness of weapons systems while firing on live targets;
- (2) the allocation procedure of assigning weapons to targets;
- (3) the inefficiency of fire when other than live targets are engaged; and
- (4) the effect of terrain on limiting the firing activity and on mobility of the systems.

The first three effects are included in the attrition coefficient as

$$A_{ij}(r) = \alpha_{ij}(r)e_{ij}(r)I_{ij}(r)$$

$$B_{ji}(r) = \beta_{ji}(r)h_{ji}(r)K_{ji}(r)$$

where

$\alpha_{ij}(r)$  = the attrition rate--the rate at which an individual system in the  $i^{\text{th}}$  Blue group destroys live  $j^{\text{th}}$  group Red targets at range  $r$  when it is firing at them,

$e_{ij}(r)$  = the allocation factor--the proportion of the  $i^{\text{th}}$  Blue group systems assigned to fire on the  $j^{\text{th}}$  group Red targets which are at range  $r$ .

$I_{ij}(r)$  = the intelligence factor--the proportion of the  $i^{\text{th}}$  group firing Blue weapons allocated to the  $j^{\text{th}}$  Red group which are actually engaging live  $j^{\text{th}}$  group targets at range  $r$ .

Similar definitions exist for components of the Red attrition coefficient,  $B_{ji}$ . The intelligence factor has not been considered in any applications to date, i.e.,  $I_{ij} = 1.0$  for all  $i, j$ .

The terrain is incorporated in the model as if it were a map with digitized properties of concealment (line-of-sight),<sup>1</sup> cover, terrain roughness, etc. associated with each or pairs of locations. Values of the attrition rates ( $\alpha_{ij}$ ,  $\beta_{ji}$ ) at any time during the battle are determined from basic weapon performance descriptors which are interrelated in attrition-rate submodels. For any weapon-target pair, the submodels determine the mean time which will be required for the firing system to kill the target system, neglecting the possibility that the firer will be killed during this period, but taking into account all the other conditions of battle (including exposure, movement, concealment, suppression, etc.). The attrition rate is then taken to be the reciprocal of this mean time to destroy the target. The attrition rate is discussed below.

### 2.1.1. The Attrition Rate

Basic to the differential model or theory of combat is the attrition rate, which is the rate at which a weapon system can destroy live targets when it is firing at them. In the classical Lanchester theories, the attrition rate has been assumed constant or state-dependent (dependent on the numbers of surviving Red and Blue forces). The inability to obtain, other than by hindsight, a satisfactory estimate of the attrition rate for future engagements has limited the use of classical Lanchester theories for planning. However, [Bonder and Farrell, 1970] have developed such a means of predicting attrition rates for a wide-spectrum of weapon systems.

---

<sup>1</sup>An exception to this statement is the version of the differential models in the VECTOR series of theater campaign models, which incorporates random variations in terrain line-of-sight.

In their developments, the attrition rate is assumed to be dependent on a multitude of physical parameters of a weapon system which describe its capabilities in such areas as acquisition, firing accuracy, delivery rate, and warhead lethality. This dependence gives rise to two distinct variations in the attrition rate--variation with range to the target and chance variation at any specific range.<sup>1</sup> A mathematical structure of heterogeneous-force combat which includes the range and chance variations explicitly cannot be analytically solved with existing mathematical techniques. For this reason we have suppressed the explicit chance variation and used average attrition rates. In this formulation we can consider the range variation of the attrition rate explicitly and somewhat independently of the chance variation at each specific range to the target.

Initially, the attrition rate at each range was defined to be the arithmetic mean or expected value of the attrition-rate random variable. [Barfoot, 1969] suggested that a more appropriate definition of the attrition rate, when a single value is used at a specific range, is the harmonic mean of the attrition-rate random variable. The appropriateness of this definition for use in the differential equation model of combat is seen below.

Consider a homogeneous-force battle in which the initial numbers of Blue and Red forces are sufficiently large so that neither is totally annihilated. Each Blue weapon system is engaged in a renewal process of attriting targets, i.e., the times between kills are independent and

---

<sup>1</sup>For clarity of discussion, variations in the attrition rate due to changes in target posture, environmental effect, etc., which can be included in the model, are not presented.

identically distributed random variables. From Blackwell's theorem [Parzen, 1962] we have

$$\lim_{t \rightarrow \infty} \Pr[\text{renewal in } (t, t + dt)] = \frac{dt}{\mu},$$

where

$\mu$  the expected interrenewal time.

Therefore, the expected number of Red kills in  $(t, t + dt)$  is

$$E[\text{number of Red kills in } (t, t + dt)] = \frac{m dt}{\mu}. \quad (1)$$

The differential equation homogeneous-force model of combat states that

$$\begin{aligned} dn &= E[\text{number of Red kills in } (t, t + dt)] \\ &= \alpha m dt. \end{aligned} \quad (2)$$

Comparison of (1) and (2) suggests that  $\alpha$  be defined as  $1/\mu$ . More generally, the definition of the attrition rate to use (for a specific range) in the differential equation structure of heterogeneous-force combat is

$$\alpha_{ij}(\text{at range } r) \stackrel{\text{def}}{=} \frac{1}{E[T_{ij}|r]},$$

where

$E[T_{ij}|r]$  = the expected time for a single Blue system of the  $i^{\text{th}}$  group to destroy a passive  $j^{\text{th}}$  group Red target, given the target at range  $r$ .



This definition for an average value of the attrition rate at range  $r$  is equivalent to the harmonic mean of the attrition rate when it is viewed as a random variable at range  $r$ . This definition also leads naturally to defining the range variation of the attrition rate as the variation in the reciprocal of  $E[T_{ij}|r]$  as the range to the target changes. The range variation is called the *attrition-rate function* and is denoted by  $\alpha_{ij}(r)$ , as used in the differential equation structure of combat.

Based on the above discussions, research on attrition rates has been concerned primarily with the development of *time to kill* probability distributions and their expected values for a spectrum of weapon systems. The distribution for the time-to-kill random variable is developed by consideration of the number of rounds expended to achieve the kill. Thus, the amount of ammunition resources expended can be obtained directly for a specific combat activity. Essentially, what is done is to take the physical process of the duel (which is basic to Monte Carlo simulations) and model the dynamics of this process mathematically.

To ensure that the attrition rates developed are general, a taxonomy of weapons systems that is not dependent on physical hardware characteristics (such as caliber) was developed. Rather, the taxonomy reflects characteristics of weapons systems that would affect the methods used in predicting the attrition rates.

The taxonomy is shown in figure 2.1. Weapon systems are first classified by their lethality characteristics as having either impact-to-kill mechanisms or area-lethality effects. Within each of these categories, we have found it useful to further classify weapon systems on the basis of

FIGURE 2.1: WEAPON SYSTEM CLASSIFICATION FOR THE  
DEVELOPMENT OF ATTRITION RATES

LETHALITY MECHANISM:

1. IMPACT
2. AREA

FIRE DOCTRINE

1. REPEATED SINGLE SHOT:
  - \* a) WITHOUT FEEDBACK CONTROL OF AIM POINT
  - \* b) WITH FEEDBACK ON IMMEDIATELY PRECEDING ROUND (MARKOV FIRE)
  - c) WITH COMPLEX FEEDBACK
2. BURST FIRE:
  - \* a) WITHOUT AIM CHANGE OR DRIFT IN OR BETWEEN BURSTS
  - \* b) WITH AIR DRIFT IN BURSTS, AIM REFIXED TO ORIGINAL AIM POINT  
FOR EACH BURST
  - c) WITH AIM DRIFT, RE-AIM BETWEEN BURSTS
3. MULTIPLE-TUBE FIRING: FEEDBACK SITUATIONS (1a, b, c)
  - \* a) SALVO OR VOLLEY
4. MIXED-MODE FIRING:
  - a) ADJUSTMENT FOLLOWED BY MULTIPLE-TUBE FIRE
  - \* b) ADJUSTMENT FOLLOWED BY BURST FIRE

---

\* Indicates that analysis of this category has been performed.

their methods of using firing information to control the system aim point and their delivery characteristics, i.e., the firing doctrine employed.

Methods have been developed [Bonder and Farrell, 1970] that allow the prediction of attrition rates for many of the weapon systems shown in the taxonomy. The first cases analyzed involved single-tube firings in which launch of a projectile occurred only after the observation of the effects of the preceding round. These are called "repeated single-shot" doctrines in our schema, and are sometimes called "shoot-look-shoot" doctrines by other analysts. Analyses have been undertaken of two subclasses: (1) those in which no use is made of information obtained from observations and (2) those in which the observations are treated distinctly depending on whether they are a hit or a miss, leading to different types of correction in aim point for these two cases. This subclass is called "Markov fire." A completely general time-to-kill probability distribution for Markov fire systems has been developed. Weapon system parameters that are included explicitly in the distribution are shown in figure 2.2. Methods of predicting these parameters from basic hardware considerations are well known.

The more complex doctrines involving "multiple-tube firings" and "burst fire," have been analyzed separately. These are classes of systems for which the projectiles may be launched before observation of previous round effects. Burst-fire cases analyzed include those in which rounds are all identical with respect to accuracy (no drifting or controlled alteration of the aim point) and those in which the accuracies of rounds within a burst vary, but the bursts are resighted to the same aim point.

their methods of using firing information to control the system aim point and their delivery characteristics, i.e., the firing doctrine employed.

Methods have been developed [Bonder and Farrell, 1970] that allow the prediction of attrition rates for many of the weapon systems shown in the taxonomy. The first cases analyzed involved single-tube firings in which launch of a projectile occurred only after the observation of the effects of the preceding round. These are called "repeated single-shot" doctrines in our schema, and are sometimes called "shoot-look-shoot" doctrines by other analysts. Analyses have been undertaken of two subclasses: (1) those in which no use is made of information obtained from observations and (2) those in which the observations are treated distinctly depending on whether they are a hit or a miss, leading to different types of correction in aim point for these two cases. This subclass is called "Markov fire." A completely general time-to-kill probability distribution for Markov fire systems has been developed. Weapon system parameters that are included explicitly in the distribution are shown in figure 2.2. Methods of predicting these parameters from basic hardware considerations are well known.

The more complex doctrines involving "multiple-tube firings" and "burst fire," have been analyzed separately. These are classes of systems for which the projectiles may be launched before observation of previous round effects. Burst-fire cases analyzed include those in which rounds are all identical with respect to accuracy (no drifting or controlled alteration of the aim point) and those in which the accuracies of rounds within a burst vary, but the bursts are resighted to the same aim point.

All present analyses have been based on fixed-length bursts. The complex case in which bursts are re-aimed on the basis of observation has not been analyzed.

Analyses have been conducted of multiple-tube firing cases, and it has been determined that the attrition rate for both volley and salvo fire may be represented by the same formulae. The method developed considers a weapon system which, perhaps not knowing the exact location of targets, fires indirectly into an area with a projectile that delivers damage-producing effects over part of the area. Parameters included in the method are shown in figure 2.3. Each of these parameters can be predicted from basic hardware characteristics of weapons systems and targets.

## *2.2 Battalion Level Models and Fire Support Analyses*

VRI has applied differential models of battalion task force level combat in a wide variety of studies. The computational efficiency of the models and the high resolution present have been proved valuable in analyzing such topics as the role of attack and scout helicopters, anti-tank and tank weapon systems, the effect of terrain line of sight variations, and the effectiveness of the cannon launched guided projectile (CLGP). The full extent of systems, processes and environmental variables that have been incorporated are illustrated in figures 2.4, 2.5, and 2.6 respectively. However, in analysis of fire support mix questions, the differential models described above are not used in a stand alone mode but form a component of larger models. The nature of this approach and the rationale for its use are described in the next section.

FIGURE 2.3: FACTORS CONSIDERED IN ATTRITION RATE FOR  
INDIRECT, AREA-FIRE WEAPONS

WEAPON AIMING AND BALLISTIC ERRORS

TARGET LOCATION ERRORS

WEAPON FIRING RATE

VOLLEY DAMAGE-PATTERN RADIUS

TARGET DISTRIBUTION

TARGET RADIUS

TARGET POSTURE

PROBABILITY THAT THE TARGET IS DESTROYED GIVEN IT IS  
COVERED BY DAMAGE PATTERN

FIGURE 2.4: THE DIFFERENTIAL MODELS: SYSTEMS INCLUDED

TANKS, INCLUDING SECONDARY ARMAMENT

APC'S, INCLUDING MULTIPLE ARMAMENT SYSTEMS

ANTI-TANK GUNS AND MISSILES

ASSAULT GUNS

HEAVY MACHINE GUNS

MORTARS

RIFLE SQUAD WEAPONS, INCLUDING

LIGHT AND MEDIUM MACHINE GUNS

GRENADE LAUNCHERS

MIXED-MODE WEAPONS

RIFLES

CONVENTIONAL, ICM, AND LASER-GUIDED ARTILLERY

ATTACK HELICOPTERS WITH

AUTOMATIC WEAPONS

ROCKETS

COMMAND-GUIDED MISSILES

SELF-GUIDED MISSILES

LASER-GUIDED MISSILES

ROCKET OR MISSILE ARTILLERY

FIXED-WING TACTICAL AIRCRAFT WITH CONVENTIONAL OR ADVANCED ORDNANCE

AIR DEFENSE GUNS AND MISSILES

LAND MINES, INCLUDING SCATTERABLE MINES

JEEP AND TRUCK MOUNTED WEAPONS

LASER DESIGNATORS

FIGURE 2.4: THE DIFFERENTIAL MODELS: SYSTEMS INCLUDED

*(Concluded)*

TARGET ACQUISITION SYSTEMS, WHETHER GROUND OR AIR BASED, INCLUDING OPTICAL  
AND OTHER ELECTROMAGNETIC SYSTEMS AND SEISMIC, AUDIO, AND OTHER  
SYSTEMS  
SMOKE OR OTHER OBSCURANT AEROSOL, HOWEVER DELIVERED



FIGURE 2.5: THE DIFFERENTIAL MODELS: PROCESSES MODELLED

ACQUISITION, "SERIAL" OR "PARALLEL," INCLUDING FALSE ACQUISITIONS,  
 ACQUISITIONS OF DEAD TARGETS, AND MIS-IDENTIFICATION (AND LOSS OF  
 ACQUISITION)

TARGET SELECTION, INCLUDING CRITERIA FOR THE ACCEPTANCE OF LOW-PRIORITY  
 TARGETS (AN APPROXIMATE MINIMAX TARGET SELECTION PROCESS IS AVAILABLE  
 IN ADDITION TO DESCRIPTIVE MODELS)

AIMING, ROUND SELECTION, AND MODE-OF-FIRE SELECTION, INCLUDING FIRE  
 ADJUSTMENT PROCESSES

FIRING, DIRECT AND INDIRECT: SINGLE ROUNDS, VOLLEY, AND BURST; ADJUSTED  
 AND UNADJUSTED; BALLISTIC ORDNANCE, COMMAND-GUIDED ORDNANCE, SELF-  
 GUIDED ORDNANCE, ILLUMINATION-GUIDED ORDNANCE; ETC.

ORDNANCE LETHALITY, IMMEDIATE OR DELAYED, AGAINST WEAPON SYSTEM HARDWARE  
 OR CREW, INCLUDING MULTIPLE DAMAGE STATES (WHICH MAY INVOLVE DAMAGE  
 TO ONLY ONE COMPONENT OR SUB-SYSTEM OF THE WEAPON SYSTEM, SUCH AS A  
 MOBILITY KILL OR A PARTIAL FIREPOWER KILL)

MANEUVER

DELIBERATE DETERMINISTIC OR STOCHASTIC USE OF LOCAL TERRAIN OR VEGETATION  
 FOR COVER AND CONCEALMENT, INCLUDING BUT NOT LIMITED TO SUPPRESSION  
 BY ARTILLERY OR DIRECT FIRES

COMMUNICATION OF TARGET ACQUISITION INFORMATION BETWEEN WEAPON SYSTEMS

DAMAGE RECOVERY, INCLUDING RE-MANNING OF A WEAPON SYSTEM WHICH HAS SUFFERED  
 A CREW KILL

MINEFIELD ENCOUNTER, INCLUDING INITIAL ENCOUNTER ATTRITION, ATTRITION  
 DURING REORGANIZATION (IF ANY), CLEARING OR PASSAGE TACTICS DECISION,

FIGURE 2.5: THE DIFFERENTIAL MODELS: PROCESSES MODELED

*(Concluded)*

MANEUVER ALTERATIONS FOR CLEARING, PASSAGE, OR ATTEMPTED BYPASSING,  
AND ATTRITION BY MINES DURING PASSAGE, CLEARING, ETC.

AEROSOL GENERATION AND CONSEQUENT ACQUISITION AND ILLUMINATION

ENVIRONMENTAL DEGRADATION

## FIGURES 2.6: THE DIFFERENTIAL MODELS: ENVIRONMENTAL SUB-MODELS

DETERMINISTIC TERRAIN AND VEGETATION

DETERMINISTIC ATMOSPHERIC TRANSMISSIVITY AND ABSORPTION

(DETERMINISTIC METEOROLOGICAL VISIBILITY)

DETERMINISTIC LINES-OF-SIGHT (FUNCTIONS OF DETERMINISTIC TERRAIN AND  
VEGETATION)

STOCHASTICALLY DESCRIBED TERRAIN AND/OR VEGETATION

DETERMINISTIC TERRAIN AND/OR VEGETATION DETERMINED AS A SAMPLE FROM A  
STOCHASTIC TERRAIN

OBSCURATION PRODUCED BY COMBAT ACTIVITIES (MOVEMENT AND FIRING, ETC.)

STOCHASTIC OBSCURATION

DETERMINISTIC BACKGROUNDS AND ILLUMINATIONS AS FUNCTIONS OF LOCATION

STOCHASTIC BACKGROUNDS

DETERMINISTIC CLOUD HEIGHTS AND LOCATIONS

STOCHASTIC CLOUDS

### 3.0 CURRENT METHODOLOGY

The differential models described in the previous section serve primarily as high resolution models of direct fire combat in maneuver unit engagements. Although forms of differential models have been used to calculate the results of brigade or division level combat in a single set of equations, we at VRI prefer to model combat at the brigade or division level as consisting of a set of company or battalion level actions that occur in sequence and/or in parallel, with each action modeled by means of an appropriate differential model. There are various reasons behind this approach, among the most important are the need to adequately represent command and control of maneuver units in a division or brigade engagement and the need to represent the dynamic changes in the demands on and availability of components of the combat process of which fire support is perhaps the best example. Accordingly, a series of models have been developed by VRI which are differential models in the sense that direct fire engagements are modeled via high resolution differential models of company or battalion level combat within an overall structure that incorporates deterministic or expected value models of other battlefield processes. In this fashion a high degree of resolution is achieved in representing the spatial and temporal interactions between the entities and processes which comprise a battle or a campaign.

In conducting analyses of fire support mixes the inclusion of spatial and temporal interactions is critical. Due to the long-range effectiveness of the weapon systems in a fire support mix, the impact of fire support is not restricted to a single location on the battlefield; the targets of a battery or an aircraft sortie could at one instant be an engaged threat

and at the next instant a weapon system or concentration a substantial distance behind the FEBA. The impact of fire support may be immediate in the case of final protective fires, of intermediate duration in the case of suppression of threat air defense systems or delay on a reserve maneuver unit moving to the front or of long duration in the case of the destruction of supplies or transportation resources.

The fire support system must simultaneously deal with spatially separated targets of possibly critical importance; in certain situations the number of targets requiring fires may exceed the number of fire support resources available. Furthermore, the fire support mix is itself subject to attrition and suppression by counter-battery fires in the case of artillery; by air defense systems in the case of close air support, both fixed wing and helicopter. Based on these characteristics of the fire support process, it can be argued that analyses of fire support mixes must at some point include the examination of the contribution of the mixes to engagements of at least brigade and more properly division size, of sufficient duration to permit assessment of the impact of the long term effects of fire support missions and possible threat responses directed at the fire support resources.

### *3.1 General Approach*

VRI has constructed a number of models of combat at the brigade, division and theater levels which have as their basis differential models of engagements between maneuver units. Our general approach to modeling combat in these scenarios has two basic components: the concept of the state space and the concept of process models. The state space consists

of those variables whose values completely describe the battlefield at any given instant, together with variables whose values suffice to permit the calculation of the future course of combat. In the first category are variables describing the numbers and locations of different weapon systems, their organization, their activities, etc., while the second category includes variables describing such things as target lists, plans and intentions, etc. The state space is frequently thought of as consisting of the information that would constitute a "snapshot" of the battlefield, together with information describing intended behavior or courses of action. (Historical information is also included as input to tactical decision making.)

To calculate the changes in the values of elements of the state space that occur as a consequence of activities on the battlefield process models are used. The differential models of maneuver unit combat are the foremost example of process models, others include supply and ammunition consumption, target acquisition, tactical decision making, air to ground firepower, etc. Inputs to process models include not only state variable values, but also performance and/or environmental data. The key characteristic of VRI's approach to both state space and process models is that, insofar as is possible, state space variables and data are based upon experimentally or doctrinally verifiable quantities. Such concepts as "firepower score" are not used.

Within the state space/process model approach to modeling combat, it is possible to achieve efficiency in operation without substantial degradation of resolution by logically selecting state variables that can represent aggregates rather than single entities and by calling upon process

models to calculate the effects of activities with a frequency consistent with the impact of these activities on the spatial and temporal interactions on the battlefield. Thus for example, the location of batteries of artillery (rather than single tubes) is sometimes assumed to be described by a distribution, and the effects of fire support on rear area targets may be calculated less frequently than the effects of fires in a maneuver unit combat. The governing factor in selecting either aggregation policies or intervals for calculation of effects is the necessity of representing the possibly simultaneous interactions between entities and activities on the battlefield.

### *3.2 Fire Support Processes*

In the context of the general approach to modeling large scale combat activities, two major components must be formulated to properly represent fire support, namely state space variables and process models. The state space variables describe the components of the fire support mix, their status, activities and intentions, while the process models describe the activities of the fire support mix and its interactions with the total combat process. Needless to say, the attrition and suppression of elements of a fire support mix are critical to any analysis of the course of a battle or campaign in which such a mix is employed. In this paper we will discuss these problems only briefly and then from the point of view of the causative agent rather than the target.

Typically we have described the elements of a fire support mix in large scale models in terms of state space variables which represent numbers, location, status and activity. In some cases where location is

not sufficient to indicate ownership, a further descriptor is used, e.g., to describe the use of corps artillery in support of a specific division or brigade. It is possible in the models to represent variable numbers of different types of artillery, fixed wing aircraft and helicopters, together with different ordnance loads. Aggregation at the battery or flight level is generally used, and positional information is represented in terms of distributions over areas associated with specified levels or organization, i.e., division or brigade artillery, or the position of a flight of aircraft on the ingress, target area or egress portion of the flight path. Provision has been made to represent suppression of artillery batteries and to indicate unavailability due to movement.

A second set of state space variables is used to describe targets for fire support missions. Inventories are maintained of targets by type, location and activity. Target types refer to both composition and behavior. A number of generic categories is supplied within the model and the user provides information of the elements making up the target, their deployment and their behavior. This information constitutes data used as input to both target detection and attrition processes.

The final set of state space variables directly associated with fire support is that which describes the target acquisition resources. Again, inventories of these resources are maintained by type and location. Where appropriate location is implicitly used to define the unit to which a target acquisition resource reports its acquired targets. Target acquisition resources are subject to attrition and suppression.

Three different process models are directly associated with fire support activities:



- (1) target acquisition,
- (2) target allocation, and
- (3) delivery.

These models are used, as described above, to calculate dynamic changes in state space variable values and thus to represent interactions between elements of the fire support mix and other entities and activities on the battlefield. The target acquisition model, based on the classic non-homogeneous Poisson process model of detection, produces the expected number of acquired targets characterized by type, location and type of resource making the acquisition. The expected number of targets acquired is a function of target types and numbers, range between target and sensor, line of sight properties, target composition and behavior, environment and the duration of the search period. Provision has been made for sensors which operate intermitantly and for sensors that "hold" an acquired target after detection. Target reports are generated from target acquisitions to represent non-identification and the reporting of targets is delayed to reflect processing and communication.

The output of the target acquisition process model is an array of the expected number of acquired targets by type and location. This array which changes dynamically as further acquisitions are reported and as missions are fired, is the primary input to the target allocation process model. This model represents the decision making process which assigns targets to elements, e.g., batteries and flights, of the fire support mix. It has been our practice at VRI to base the logic of target allocation upon current doctrine rather than to attempt in any way to "optimize" the fire

support process. The target allocation process model is extremely flexible in this regard and logic has been designed which incorporates such variables as the number, type and location of reported targets, the lengths of time since acquisition, the CEP of the reporting sensor, the number and type of fire support resources available, current missions and ammunition and/or fuel constraints. The outputs of the rule specify which elements of the fire support mix fire on which targets and also specify the type and amount of ordnance to be delivered. A further delay representing processing and communication is assessed in the target allocation model.

The calculation of attrition and initiation of suppression is accomplished by the delivery model, which also schedules the attrition to account for delays caused by battery preparation and flight times. The quantity calculated is the expected amount of attrition as a function of actual target composition (as distinguished from the reported target) and amount and type of ordnance delivered. As the duration of the period between acquisition and delivery is available in the model, the effects of target movement, target location and delivery efforts are incorporated into attrition calculations. Suppression is generally represented by a decrease in firing rate, ranging from complete suppression to a fraction of the normal rate of fire. The duration of suppression is a function of the time since last ordnance delivery.

### *3.3 Advantages of the Differential Models*

As discussed earlier, it is our belief that combat models at the brigade or division level are necessary for fire support mix studies, and

that these models must be capable of representing the complex spatial and temporal interactions that take place in combat. The differential based models in which maneuver unit combat is modeled by means of differential methods and other modeling techniques are used where appropriate to represent other battlefield activities, appear to offer several advantages. The first of these is the high resolution available. Without exception the methods used in the differential based models provide greater resolution than any other deterministic method. The only other methodology with which a comparable resolution is possible is that of a highly detailed Monte Carlo simulation. It can be argued that this latter method is more expensive in terms of both development and use, and that it poses problems in terms of transparency, i.e., it may be difficult to connect cause and effect using the Monte Carlo approach. In addition to efficiency and transparency, the differential models have proven to be remarkably flexible and can be quickly and easily modified to reflect new weapon systems and tactics, or to increase or decrease resolution as required, by aggregation or disaggregation of entities, processes or time.

There remain, of course, problems in differential models but these are for the most part common to all combat models. These problems might be best described as those associated with battlefield processes which are not adequately understood. Suppression is one such process; the generation of target reports from target element detections is another. It is clear also that although a substantial amount of information can be obtained in the area of target allocation, actual procedures vary from organization to organization and from officer to officer. Until more detailed knowledge becomes available on these processes, approximations must be used to

represent their effects. In the mean time the sensitivity of combat results to different approximations can be determined.

#### 4.0 THE VECTOR-2 THEATER-LEVEL COMBINED ARMS SIMULATION

This section is intended to illustrate the range and scope of differential based models of combat. Perhaps the largest such model currently available, VECTOR-2, is described. It is not suggested that this model be used for fire support studies per se, but the model is an example of what can be accomplished using the differential model approach.

VECTOR-2 is a two-sided theater-level combined arms simulation model. The model is deterministic, i.e., it produces a single engagement history for theater-level engagements of durations ranging up to 100 days. The two basic elements of the model structure are the concepts of a state space and a set of process models. The state space utilized in the model consists of a set of variables which describe the current status of all elements on the theater battlefield at any given instant during the course of a campaign. In addition, the state space includes variables which contain sufficient information to predict the immediate future of the campaign. As an illustration of this concept, consider a battalion task force on the FEBA involved in direct fire combat with an enemy unit. State space variables used in VECTOR-2 to describe such a task force include:

- (1) the composition of the task force in terms of numbers of weapon systems of up to 12 types;
- (2) the deployment of the weapon systems including movement;
- (3) the proportion of the weapon systems firing on specific enemy weapon system types and groups (including the targets of organic air defense weapons);
- (4) the intention of the task force (i.e., assault on a defensive position, hold on a defensive position, delay or withdraw);

- (5) target acquisition and firing doctrine employed by weapon system type;
- (6) targets reported by the task force to the fire support coordination center;
- (7) task force perception of enemy order of battle;
- (8) commander to whom the task force commander reports,
- (9) intensity of communications from task force to fire support coordination center;
- (10) characteristics of and distance to task force objective;
- (11) weather;
- (12) mobility characteristics of terrain;
- (13) intervisibility characteristics of terrain; and
- (14) presence of attack helicopters in a fire support role.

The extent of the portion of the state space required to describe a battalion task force in direct fire combat is a consequence of VRI's philosophy of aggregating only in those processes in which significant effects and outcomes are not masked by aggregation. Rather than aggregate effects, VRI chose to include and dynamically keep track of explicit representations of force elements, environment and processes in terms of measurable physical and behavioral variables. This choice in turn makes possible the clear definition of data requirements and subsequent ease of modification of weapon system types, capabilities, and employment doctrines.

The introduction of new processes, or elements, or the enrichment of current processes or elements in the VECTOR series of models usually results in the addition of states to the state space. The selection of these states is based on the level of detail and aggregation selected for

the process introduced or enriched and the need to maintain a consistent level of detail throughout the model. Thus, for example, the addition of command and control to VECTOR-1 as part of the development of VECTOR-2 led to a different and finer representation of model time which in turn led to a much more detailed description of close air support aircraft. States were added to indicate airborne or airbase alert, specific ordnance load, ingress, egress, target area activity, and endurance. Delivery delay times were explicitly represented in the state space together with decision state variables to represent the employment of CAS in those cases where the number of sorties available did not meet missions demanded.

All interactions between elements of the combined arms forces involved on both Blue and Red sides in VECTOR-2 are reflected in changes of the values of state space variables. These changes of values are determined via process models which calculate the effects of current activities on state variables. These process models range in complexity from a single priority scheme to describe the order in which a field artillery battery engages acquired targets to a comprehensive differential model of battalion task force direct fire combat or a detailed model of the effects (attrition) produced by the delivery of ordnance by a CAS aircraft on a maneuver unit target.

Process models in VECTOR-2 were selected on the basis of a number of factors. The first of these deals with the level of detail required to adequately model the effects of the process on state space variables. The second deals with interaction between processes and may be thought of as related to timing. Activities that are interacting instantaneously must be treated simultaneously by appropriate models. Thus the evolution of the VECTOR series of theater-level models can be characterized in one

respect as involving a continual decrease in the number of processes that are decoupled from the combat viewed from the theater-wide perspective. Subject to constraints imposed by computer storage and running time, some activities whose effects are less immediate are calculated periodically, but efforts have been directed toward the objective of including all interactions simultaneously whenever feasible.

The description of VECTOR-2 that follows is intended to illustrate the conceptual structure of state space and process models. The description is organized as follows. Categories of state variables are discussed first, including representation of

- (1) battlefield, environment and time,
- (2) forces,
- (3) supplies, and
- (4) plans and intentions.

Major process models incorporated in VECTOR-2 are then described, including:

- (1) command, control, and communication;
- (2) intelligence/target acquisition;
- (3) firepower;
- (4) logistics and supply; and
- (5) movement.

Following the description of the model, some experience with VECTOR-0 and VECTOR-1 is described.



#### 4.1 *Categories of State Variables*

##### 4.1.1 Representation of Battlefield, Environment, and Time

The theater-level battlefield in VECTOR-2 is divided into ten sectors roughly corresponding to areas in which one or more US Army corps might be deployed. Within a sector a further division of the battlefield is made on the basis of military characteristics, into areas referred to as combat arenas. Each sector has a fixed (for a particular model execution) number of ribbons of combat arenas. The arenas themselves vary in terms of width and depth. The width of an arena corresponds approximately to the defensive front of a US Army battalion and is a function of terrain characteristics. The depths of arenas are determined by the existence of natural or manmade features which constitute objectives for combat forces; the degree of correlation between objectives in adjacent arenas is specified as are the characteristics of the objectives, e.g., river, urban area, or hill. Defensible positions are represented internal to a combat arena. An arena may contain any number of defensible positions spaced at equal distances in depth subject to the constraint that the positions are separated by the maximum range of direct fire weapons.

Within an arena both trafficability characteristics and intervisibility characteristics are assumed to be homogeneous. The state variables for trafficability and intervisibility for an arena can each take on one of five values. Weather is represented in VECTOR-2 by a state variable for each sector. This state variable may also take one of five values. Consequently, 125 different environments which impact on combat are represented in VECTOR-2.

The representation of time in VECTOR-2 reflects the need to calculate simultaneously the effects of activities which interact instantaneously, and at the same time to calculate efficiently the effects of activities for which exact timing is not critical. Within a sector of the theater, time is essentially continuous over a model time period (a user-selected duration, usually 12 or 24 hours). This continuity is accomplished by combinations of a time step procedure, for those activities in which major changes can occur in intervals of approximately 15 to 30 seconds duration, and an event scheduling procedure, which schedules the calculation of effects which are subject to delay in reality (e.g., delivery of fire support), and the calculation of effects of activities periodically when the effects of those activities are not immediate (e.g., arrival and distribution of supplies or replacements at corps level).

#### 4.1.2 Representation of Forces

For each side the model considers maneuver forces at the FEBA, maneuver forces in reserve, artillery forces, attack helicopters, air defense artillery forces, tactical fixed wing air forces and services support forces. Maneuver units, both at the FEBA and in reserve, can contain a user-selected number of weapon system types. (Demonstration runs at VRI will employ two types of tank systems, three types of anti-tank systems, armored personnel carriers, infantry with heavy automatic weapons, mortars or similar area fire weapons, infantry with basic rifle squad weapons, two types of air defense weapons systems and attack helicopters.) Artillery forces will contain up to four weapon system types together with associated personnel,

attack helicopters will be of one type with personnel. Air defense artillery forces can contain up to six different types and provision is made for eight types of fixed wing aircraft with ten different ordnance loads. Airbases of three different types are included in the model, together with shelters and support personnel. Service support forces consist of personnel only. The model continually keeps track of the number of weapon systems and personnel for both Red and Blue by maneuver unit. The number of weapon systems are separately retained by type for artillery forces, attack helicopter forces, air defense artillery forces, tactical air forces and service support forces and are maintained according to that element of the command and control hierarchy to which they belong or are attacked. Also retained in this fashion are target acquisition resources: up to fourteen types ranging, for example, from forward observers to early warning radars.

#### 4.1.3 Supplies Represented

Supplies of the following kinds are represented separately in the model: ammunition for each maneuver unit weapon system type, ordnance (in user-specified categories) for fixed wing aircraft, aviation gasoline and associated POL (for fixed-wing aircraft and attack helicopters), POL for ground systems, mines and a user-specified category, other supplies. Ammunition is assigned (and bookkept by type) to units such as individual tactical air forces, to sector (corps) stores, and to theater stores. POL is similarly assigned to individual battalion task forces, to air bases, sector stores and theater stores. The user-specified "other supplies" are assigned to sector stores and to theater stores.

#### 4.1.4 Plans and Intentions

Among the most significant state variables within the VECTOR-2 state space are those which describe the plans and intentions of the elements of the ground forces command and control hierarchy, from theater to battalion task force level, and the air force command and control hierarchy at theater and flight levels. Essentially, plans and intentions are set by tactical decision rules which correspond to decisions made at various levels in the command and control hierarchy. Thus the plans and intentions of one unit (e.g., a division) are used to organize subordinate units for combat, to assign missions to these subordinate units and then to coordinate units. Plans and intentions in the air portion of the model are used as input to those decision processes which deploy aircraft to air bases, select missions and organize aircraft for those missions, and govern the tactical behavior of fixed wing aircraft. It is as a consequence of the interaction between the plans and intentions of Red and Blue that activities occur in the VECTOR-2 model.

In the preceding section the significant elements of the state space of VECTOR-2 were briefly described. Each of these states are variables in the model which may change with the passage of time. At any instant the values of the variables represent a picture or "snapshot" of the battlefield and activities underway at that time. In addition to providing this instantaneous picture, the state space is so structured as to provide information which cause changes and the associated process models are now described.

## 4.2 Processes in VECTOR-2

### 4.2.1 Command, Control and Communication

In VECTOR-2 a command and control hierarchy is explicitly represented. As a consequence, for any unit both superior and subordinates are identified, together with the decisions which are made by the commander of the unit. Both the command and control hierarchy and the above decisions may differ between Red and Blue or within sectors on Red and Blue sides.

The conceptual basis selected for the  $C^3$  process model in VECTOR-2 is that of a feedback control system. In such a system a desired state of the world, or reference input, is compared to an observed state of the world, or feedback signal. As a consequence of this comparison, control elements are applied when necessary to produce, insofar as is possible, an observed state of the world which conforms to the desired state. In a broad sense control is necessary when the system is disturbed by undesired input. Further complications arise because of time lags and/or inaccuracies and/or omissions in the feedback signal and because of time lags in the application of control elements.

The analogy between the feedback control system and the  $C^3$  process proceeds as follows. A commander is assigned a mission corresponding to a reference input or desired state of the world, e.g., to hold a position, to destroy an enemy formation or to assault and take a position. The commander examines the "system" in which he must function based on intelligence estimates of the enemy, estimates of the capabilities of his own forces and other factors which could be said to make up the commander's perceived state of the world, perceived because information, about the

enemy in particular, may be incomplete and/or inaccurate. Based upon a comparison of the mission and current situation, the commander allocates resources to accomplish the mission or to bring the perceived state of the world into agreement with the desired state of the world.

The structure described above is sufficiently broad to be applied to  $C^3$  at all levels of the  $C^3$  hierarchy; that is, from squad to theater level. The significant differences between levels may be related primarily to the size of the system considered by the military decision maker at any specific level, the resources available to that decision maker, and the time frame within which the decision maker can act to bring resources to bear within the system. A further characteristic of the  $C^3$  process that should be noted is that application of control elements at one level results in the specification of a desired state of the world for  $C^3$  elements subordinate to that level.

A diagram of the  $C^3$  process viewed in the context of a feedback control system is given in figure 4.1. Note that a similar but not necessarily identical structure will exist for the enemy. For purposes of illustration consider a specific level or echelon within the  $C^3$  hierarchy. At that echelon missions are assigned via a communications system by a higher echelon. The military decision maker compares the mission with knowledge of the enemy (intelligence estimates), knowledge of the combat environment (terrain, weather, etc.) and knowledge of his own forces and then assigns missions to his subordinate elements. These assignments are communicated via the communications system. When and if combat begins, the decision maker receives, again via the communications system, information from his own forces as to their progress toward completing the mission as

**Proposed:**



well as intelligence estimates of enemy action which is stalling or contributing to that progress. Upon receipt of this information the decision maker can utilize his own resources or request other resources to make the dynamics of the combat correspond to his desired goals, i.e., as a consequence of feedback, control elements are applied.

In VECTOR-2 the desired state of the world, or reference input in control theory terminology, will consist of an assigned mission or, in the case of dynamic combat, of a set of parameter values which describe that combat. The perceived state of the world will consist of three elements. Knowledge of the enemy will be provided via an intelligence model. Knowledge of the combat environment, i.e., environmental parameters not subject to human control, will be provided via a data based description of the scenario. Finally, knowledge of own forces will also be available subject to delay caused by communications. Elements of information flow on feedback loops will be subject to choice by the model user; time delays associated with both decision making and implementation of decisions, i.e., the issuing of orders, will be represented. Significantly, this approach will enable the user of VECTOR-2 to examine different  $C^3$  structures and to examine the effects of incomplete or delayed information on the decision-making process carried out by these  $C^3$  structures in addition to the effects of time delays during the transmission of orders.

The primary structural component of the  $C^3$  process model in VECTOR-2 is the tactical decision rule. A tactical decision rule is a computer subroutine which models a decision-making process. With reference to the above description of the structure of the command and control process, the inputs to a tactical decision rule consist of state variables in the



model which describe the desired state of the world; for example, mission or desired supply levels at subordinate units. Also constituting inputs to the rule are state variables which describe the perceived state of the world in three subcategories: those state variables which represent intelligence estimates of enemy characteristics and strength, those state variables which describe the environment such as terrain, weather, etc. and those state variables which describe friendly resources, including strengths, dispositions and supply levels. Within the rule the desired and perceived states of the world are compared via decision logic which is in some cases augmented by a data base inherent only to the decision rule. The result of the comparison, a detection, is then implemented by changing the values of state variables which form the output of the decision rule.

The subroutine structure selected for tactical decision rules in the VECTOR model series provides a number of advantages. First, the rules can be easily altered by simply replacing a subroutine, facilitating the study of different decision thresholds or completely different decision logic (i.e., different employment doctrines). Secondly, the tactical decision rules in VECTOR-2 are inherently extremely flexible since, in addition to their role in which the values of specified state variables must be set, they can be used to incorporate new processes and effects to the model. This latter feature is a consequence of the subroutine structure in which the subroutine has access to almost all state variables in the state space. Hence if a new activity is to be investigated, rather than modify or augment the entire program, the activity can be represented in an appropriate tactical decision rule.

Before presenting a representative list of decisions modeled via tactical decision rules, it is useful to briefly outline the communications process model. Essentially the effects of communications in VECTOR-2 are represented by delays in information flow. Thus information provided to tactical decision rules, i.e., state variable values, is delayed by communications, as are the changes in state variables which are made by the tactical decision rule. The delays imposed via communications are modeled by a multi-server Markovian queueing network which links elements of the command and control hierarchy. Two classes of messages are represented, priority and non-priority, and expected delays are calculated as a function of the load on the system (which changes dynamically), the number of channels available, and the mean channel holding time required by messages between elements in the command and control hierarchy.

The mechanisms used to model tactical decision making and communications delays have been briefly discussed above. The following list contains the tactical decision rules employed in VECTOR-2.

(1) theater level

- (a) assignment of missions and objectives to sectors,
- (b) assignment of maneuver units to sectors,
- (c) assignment of non-organic air defense artillery units to sectors,
- (d) assignment of non-organic attack helicopter units to sectors,
- (f) assignment of fixed wing aircraft to airbases,
- (g) assignment of replacements and supplies to sectors,
- (h) assignment of missions and tactics to fixed wing air.

## (2) sector level

- (a) assignment of missions to subordinate units (sector to corps, corps to division, division to brigade),
- (b) assignment of missions to non-organic field artillery,
- (c) assignment of missions to non-organic air defense artillery,
- (d) assignment of missions to non-organic attack helicopters,
- (e) assignment of replacements and supplies to subordinate units (sector to corps, corps to division, division to brigade, brigade to battalion task force),
- (f) assignment of missions to and creation of battalion task forces,
- (g) assignment of minefields to battalion task force arenas,
- (h) allocation of prescheduled close air support sorties,

## (3) battalion task force

- (a) assessment of dynamic status of engaged battalion task force,
- (b) response to dynamic status of engaged maneuver units (includes non-engaged reserves),
- (c) target priority selection and fire support allocation.

It is important to note that the frequency with which tactical decision rules are applied varies according to level and need. Theater-level rules are applied once per model time period (12-24 hours are typical) as are most of the sector-level rules. Battalion task force tactical decision rules, however, can be applied as frequently as every 30 seconds and thus exercise substantial control over the activities present in combat between maneuver units. It is the battalion task force level rules which permit accurate representation of the effects of fire support, both CAS and

field artillery, reserve commitment and coordination of battalion task forces consistent with brigade and division mission.

#### 4.2.2 Intelligence and Target Acquisition

Previous theater-level combined arms combat models have treated intelligence and target acquisition implicitly, if at all. In VECTOR-2 intelligence and target acquisition are represented explicitly by means of process models. Although target acquisition is generally considered to be part of intelligence, for the purposes of modeling the two activities were considered separately in VECTOR-2. Intelligence is viewed as that process which provides to the tactical decision maker (and thus the tactical decision rules) a portion of his perception of the state of the world, namely information on the enemy and the environment. Target acquisition is viewed as that process which develops a list of targets from which missions are assigned to such elements of the combined arms team as field artillery, CAS or in the case of air defense, air defense artillery and interceptors. The two processes are discussed separately in the following description.

One may think of the intelligence model of VECTOR-2 as a transformation which produces estimates of enemy strength by type and location as a function of intelligence collection resources and the true values of enemy strength. The process model calculates estimates of enemy strength by type and location as a function of intelligence collection resources and true values of enemy strength. Provision has been made within the model for up to 14 different intelligence collection resources, each subject

to attrition if acquired and fire upon. The model is based on a general decay methodology in which the values of prior estimates of strength are combined with newly collected information to produce current estimates. The amount of new information collected is determined by process models representing each of the intelligence collection resources and explicitly reflecting the effect on the performance of these resources by such parameters as range, enemy activity, weather and length of observation period. For a particular geographic area and enemy element, separate estimates are made of attrition, arrivals, departures and resident elements.

The primary function of the intelligence process in VECTOR-2 is to provide to the tactical decision rules estimates of enemy strengths by type and location as input to the decision making process. Whereas previous theater-level models based decisions on actual values of enemy strength, VECTOR-2 considers such information only after uncertainties and delays have been introduced to represent intelligence collection and processing.

The target acquisition process in VECTOR-2 represents the activities associated with the acquisition of targets for the fire support and air interdiction elements of the combined arms force. Within the model up to 14 different target acquisition resources or sensors can be explicitly represented, ranging from forward observers in maneuver units at the FEBA to long-range air defense radars. These resources are subject to attrition if acquired and fired upon by enemy weapon systems. Targets in VECTOR-2 are defined to be natural groupings of weapon systems and personnel, e.g., an armored platoon in a maneuver unit, an air defense site, an airbase or a flight of attack aircraft. For calculation of both

acquisition and attrition, the identities, locations and compositions of these groups are maintained.

The process model which calculates the expected number of targets acquired in an interval of time includes the effects of the length of the observation period, the total number of available targets by type and location, detection rates by sensor and target type, line-of-sight characteristics, weather, target behavior and sensor deployment. Also included are the effects of downtime on sensors which are blocked for a finite period following the acquisition of a target. Outputs of the target acquisition model are lists of the expected number of targets acquired, classified by type, location, type of reporting sensor, and time of acquisition. The list of acquired targets constitutes input to a tactical decision rule representing the fire support coordination center at low levels, and in the case of interdiction mission targets, air force command and control. Acquisitions of enemy aircraft by air defense radars are passed to a tactical decision rule which represents air defense command and control. Based on doctrine, the tactical situation and the availability of resources, targets are assigned by these tactical decision rules to appropriate field artillery batteries, CAS sorties, attack aircraft missions or interceptor sorties.

#### 4.2.3 Firepower Processes

Firepower processes in VECTOR-2 describe the different mechanisms of delivering firepower and their effects upon state variables representing force composition values and supply levels. These processes may be grouped

into four categories: air-to-air, ground-to-air, air-to-ground, and ground-to-ground. Descriptions of the processes in each of these categories are contained in VECTOR-2 as submodels based on specific assumptions about the process being described. Inputs to each of these submodels are either directly measurable quantities or can be estimated from systems engineering models or more detailed combat process models.

The air-to-air firepower processes describe the interactions between escorts and interceptors, and interceptors and attack aircraft. The process model includes provision for both duels between aircraft and a mixed engagement involving M-on-N combat. Based on criteria input via tactical decision rules, attack aircraft may or may not abort missions when attacked, interceptors may or may not return to base after engagements, interceptors may attack either attack aircraft or escort aircraft, and aircraft may choose targets based on aircraft type and/or target type. The commitment of ground alert interceptors is contingent upon detection by air defense radars and appropriate rules of engagement. Outputs of the air-to-air process models include escorts continuing missions, escorts killed, escorts returning without engaging interceptors, interceptors killed by attackers, attackers killed by interceptors, attackers aborting missions and attackers completing missions. These results are produced by both mission and aircraft type.

The ground-to-air firepower processes describe the interactions of air defense artillery against enemy aircraft involved in ingress, target area activity and egress. Two types of processes are considered: the one-sided duel between air defense artillery and aircraft engaged in attacking targets other than air defense artillery sites, and the two-sided duel between

aircraft attacking air defense artillery sites and the air defense sites responding against the attackers. The former process model considers the three portions of the aircraft mission, i.e., ingress, target area activity and egress, separately while the latter process model is used in place of target area activity for engagement in which aircraft missions involve air defense suppression. Outputs of the models include aircraft surviving ingress, aircraft completing missions and aircraft surviving egress. In the case of aircraft-air defense artillery duels, both aircraft surviving and air defense artillery destroyed or suppressed are output by the model. The employment of air defense weapons organic to engaged maneuver units is represented in two ways. The use of such weapon systems against CAS aircraft is modeled via the models described immediately above, while the effects of such weapons upon attack helicopters employed as a weapon of an engaged maneuver unit is represented in the differential models of ground combat described below.

The air-to-ground firepower process describes the effects produced by the delivery of ordnance by aircraft against ground targets including, for example, resource groups present in engaged maneuver units in combat arenas, reserve maneuver units, command posts, field artillery batteries, aircraft and shelters at airbases, air defense radars and supplies.

The attrition of ground targets by air delivered ordnance is a function of aircraft type, ordnance type, weapon delivery technique, amount of ordnance delivered, target type, target activity and environment.

Individually-targeted ordnance such as smart bombs and area-targeted ordnance such as napalm or iron bombs are represented by different models in VECTOR-2. The outputs of the process model are changes in the state



variables representing the surviving numbers of weapon systems, personnel and supplies for all types of elements in the target. Permanent and repairable damage can be represented. As described above, the generation of missions for aircraft engaged in CAS or interdiction depends upon the acquisition of targets which is explicitly modeled in VECTOR-2. The employment of aircraft, in terms of organization of flights, selection of ordnance, and mission assignment is determined by tactical decision rules representing both the tactical situation and Air Force doctrine.

Ground-to-ground firepower processes in VECTOR-2 fall into two categories: first, delivery of indirect fire by field artillery batteries against engaged enemy maneuver unit targets and enemy targets behind the FEBA, and second, direct fire combat between maneuver units.

The calculation of attrition caused by the delivery of indirect fire is scheduled by tactical decision rules to reflect the delay between target acquisition and fire mission completion. The process model of attrition by indirect fire incorporates characteristics of weapon system type, ordnance type, amount of ordnance delivered, target type, target activity, environment, type of acquisition resource reporting the target and delay since the report which led to the fire mission. Two models are included in VECTOR-2 for calculation of indirect fire attrition, one representing area-targeted ordnance such as conventional field artillery and the other individually-targeted ordnance such as the cannon launched guided projectile (CLGP). As in the case of the air-to-ground firepower process model, outputs of the indirect fire process model are changes in the state variables representing the surviving numbers of weapon systems, personnel or supplies for all types of elements in the target.

The firepower (and other) processes in the assault activity between maneuver forces at the FEBA in each combat arena are computed internally, using VRI's differential models of combat. These models describe the dynamics of small unit firefights at the FEBA in great detail. The models explicitly consider different weapon system types on each side (tanks, antitank systems, mounted infantry, etc.), characteristics of these weapon systems (their firing rates, accuracy of fire, projectile flight times, lethality of the projectile), vulnerability of the target by type, firing doctrine of the weapon system (single rounds, burst fire, volley), probabilistic acquisition of targets in the firefight, allocation priorities of weapon systems to targets, maneuver of the weapon systems and the effects of terrain line of sight on acquisition and fire capabilities. The combat model can be used with any of a number of scenarios corresponding to different terrain types, force types (armor, infantry, airborne, etc.), or other situational variations. The choice of scenarios is governed by the user-specified tactical decision rules. Each scenario determines the basic terrain characteristics, initial weapon placements maneuver tactics, etc. which will be used in the evaluation of the small-unit action. The model computes attrition of weapon systems by type and personnel for the opposing units at different range steps as the assaulting unit closes to the objective (typically calculating results for periods of the order of 8 to 40 seconds). Based on an assessment of the tactical situation accomplished via tactical decision rules a force engaged in direct fire combat may break off an assault, begin a delay or withdrawal, request fire support (including final protective fires or disengagement fires) or begin implementation of a contingency plan. In

response to the assessments of the tactical situation in combat arenas, a tactical decision rule representing the higher elements of the command and control hierarchy assigns missions to fire support resources, coordinates the activities of subordinate units, and commits reserves. (Commitment of a reserve requires that the reserve unit move to engage and be subject to attrition and delay during the move). Outputs of the model include a complete description of surviving weapon systems by type and personnel at the end of the engagements.

#### 4.2.4 Supply Process

The consumption of supplies in VECTOR-2 occurs as a result of combat activities, attrition of supply dumps, and as a result of the passage of time. Ammunition consumption during direct fire combat is computed separately at each range step in the differential models based on the expected number of rounds fired to achieve the expected attrition calculated in the model. In other combat activities consumption of supplies is computed on the same basis as its associated firepower process model. For example, if the firepower process model involves completion of sorties parallel data elements give ammunition and POL expenditure per sortie. Consumption of supplies based on the passage of time is intended to simulate activities not explicitly modeled. This type of consumption is in direct portion to unit personnel and weapon system strengths. Supply inventories are maintained at unit, sector and theater levels and are subject to attrition at all levels if reported by target acquisition resources and attacked. Tactical decision rules determining plans and

intentions for maneuver units or air force units can explicitly reflect the effect of supply levels on mission assignments.

Resupply in VECTOR-2 is accomplished by means of tactical decision rules which allocate supplies by type from theater to sectors and from sector level down through the command and control hierarchy to maneuver units, combat support units and air force units. The rules consider both current activities and inventory levels and planned activities in allocating supplies. A logistics network is not explicitly represented in VECTOR-2. A concept study was performed and a model for such a network was proposed for VECTOR-2, but resource constraints resulted in the assignment of a lower priority to the network representation and its implementation was postponed. Delays in resupply activities due to logistics network capacity can be represented by means of tactical decision rules.

#### 4.2.5 Movement Process

Movement in previous theater-level models has not been explicitly represented. In particular, movement of maneuver units at the FEBA was based on the decision to move and a movement rate which depended upon the combat activity in progress. To improve the representation of the interaction between movement and other combat activities, movement is explicitly represented in VECTOR-2. The model continually updates a record of the location of maneuver units engaged in direct fire combat within combat arenas, committed reserve maneuver units moving from assembly points to the combat arenas where they will be deployed and flights of aircraft on the ingress, target area, and egress portions of

mission flight paths. The concept of FEBA movement used in previous theater-level combined arms simulations, in which the extent of FEBA movement is calculated after combat results have been determined, is not used in VECTOR-2. FEBA movement in VECTOR-2 occurs as the result of tactical decisions to advance or withdraw in combat arenas and as a result of any combat that occurs as a consequence of these tactical decisions. Inherent in the VECTOR-2 model is the separation between Blue and Red FEBAs, which coincide only when direct fire combat is taking place.

#### 4.2.6 Model Input and Output

Categories of inputs to the VECTOR-2 model include weapon performance data, tactical decision rules and data, environmental data, and initial force command and control structure, force inventory and deployment data. Outputs are provided via a report generator which can process intermediate values of state variables for analysis purposes. Outputs used in past VRI studies include:

- (1) Daily and cumulative weapon system losses by weapon type,
- (2) Daily and cumulative casualties,
- (3) Supply totals by type of supply,
- (4) Total weapon system survivors by weapon type,
- (5) Total personnel survivors in maneuver units,
- (6) Numbers of task forces, weapons, and personnel in maneuver units in reserve,
- (7) Numbers of sorties flown on each mission by each aircraft type,
- (8) For each combat arena maneuver unit (daily):

- (a) Number of weapon systems (by type), personnel and supplies,
  - (b) FEBA position,
  - (c) Activity, and
- (9) Casualties (by location) and weapons system (by type) losses by system type which inflicts the attrition.

Additional more detailed output (or special summaries) can be produced as required by the user.

#### *4.3 Prior Experience with VECTOR Series Models*

As noted above, the programming of VECTOR-2 is scheduled for completion in September, 1975. It is worthwhile however to consider the characteristics of the computer programs associated with VECTOR-0 and VECTOR-1 the predecessors of VECTOR-2 in the model development program. In the remainder of this section these characteristics are briefly described and some insights gained in use of the models are discussed.

Both VECTOR-0 and VECTOR-1 are written in American National Standards Institute (ANSI) FORTRAN and have been run on CDC 6400 and IBM 360/67 computing systems.<sup>1</sup> The models require two to three seconds of CPU time per sector per day of combat, plus additional CPU time if extensive output is desired. (Some cases have run more than five CPU seconds per sector per day when extremely detailed output was required.) Typical total running times are from 3 to 20 minutes.

VECTOR-2 is currently being implemented in ANSI FORTRAN and is expected to be operational on the CDC 6400 and IBM 370/168 computers later this year.

---

<sup>1</sup>VECTOR-1 and its Data Preprocessor are currently being converted to run on the HIS 6000 computer system at the National Military Command System Support Center (NMCSSC).

In the course of running VECTOR-0 and VECTOR-1, several methodologically significant observations have been made. Some of these are discussed in the following paragraphs.

#### 4.3.1 Attrition is Not Uniform

The various systems are not attrited in proportion to their numbers. Further, the attrition caused by a single system type is not distributed in constant proportions across other systems. Thus, this model, which examines attrition processes in detail, differs significantly from most or all others in its predicted attrition results. This difference corresponds reasonably to the difference between the detailed predictions of small-scale combat models and the hypothesized mathematical forms which have often been used for these results in larger aggregated models.

#### 4.3.2 Individual System's Effects Observable

The effects of individual system types on the outcomes of a theater-level campaign are clearly observable and bear clear relation to the input performance assumed. An example of this can be seen in the following case: a test case hypothetical combat was run where one of the two opposing forces heavily outweighed the other in armor, but the weaker armor force possessed a hypothetical high-lethality, low vulnerability anti-armor helicopter. Figures 4.2 through 4.5 show some summary results of a run with these forces. As they show if overlayed, the initial attrition of armor helicopters is severe, and can in fact cause some movement of the FEBA in favor of the weaker armor force, but as the helicopters are

FIGURE 4.2: A HISTORY OF FEBA MOVEMENT IN  
A HYPOTHETICAL CAMPAIGN

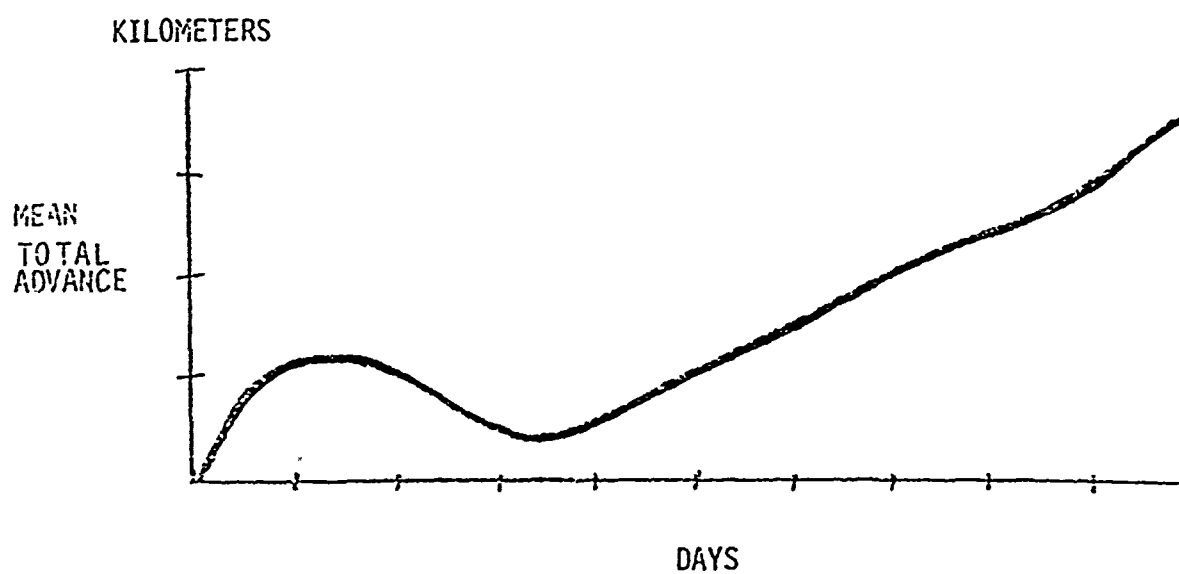




FIGURE 4.3: A HISTORY OF ATTACK HELICOPTER SURVIVORS  
IN A HYPOTHETICAL CAMPAIGN (SIDE 1)

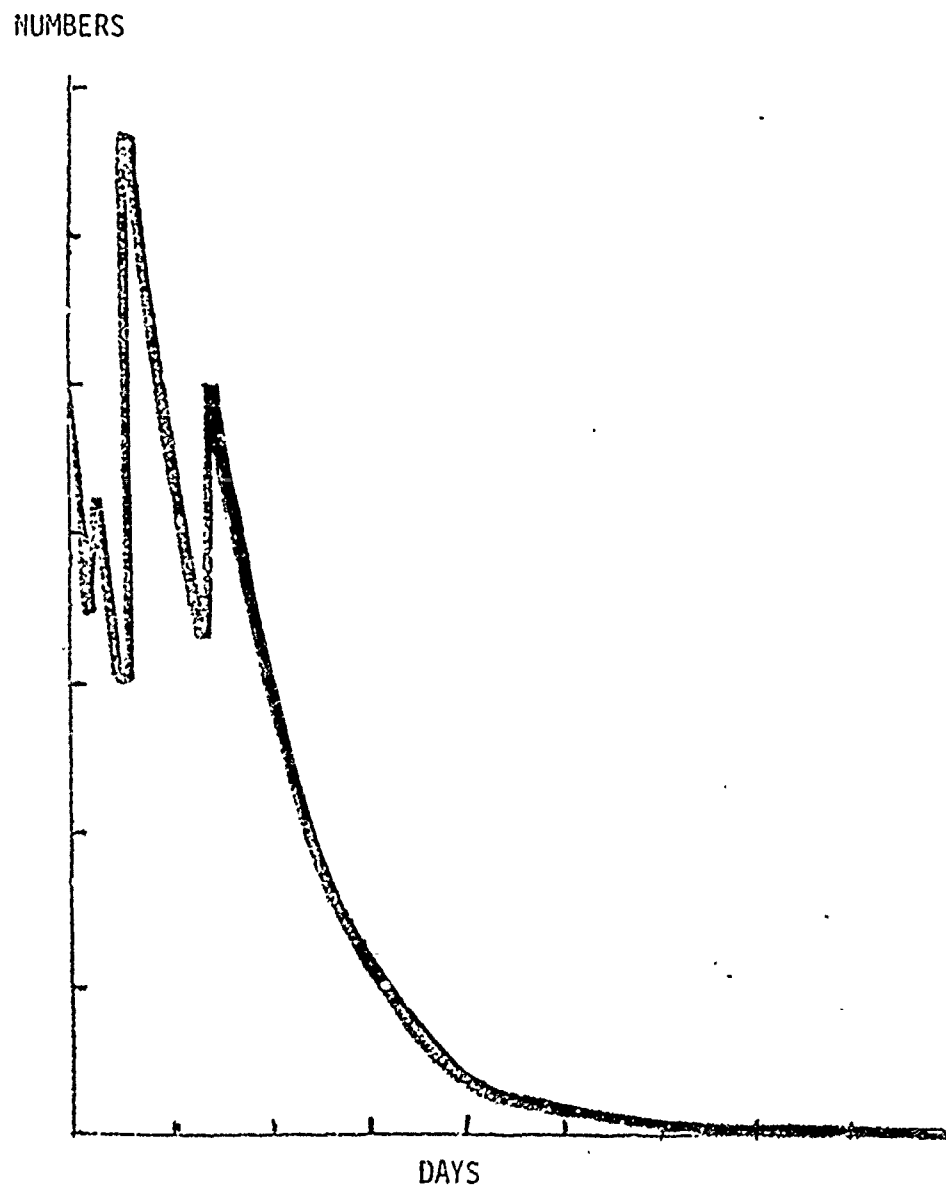


FIGURE 4.4: A HISTORY OF TANK SURVIVORS IN  
A HYPOTHETICAL CAMPAIGN

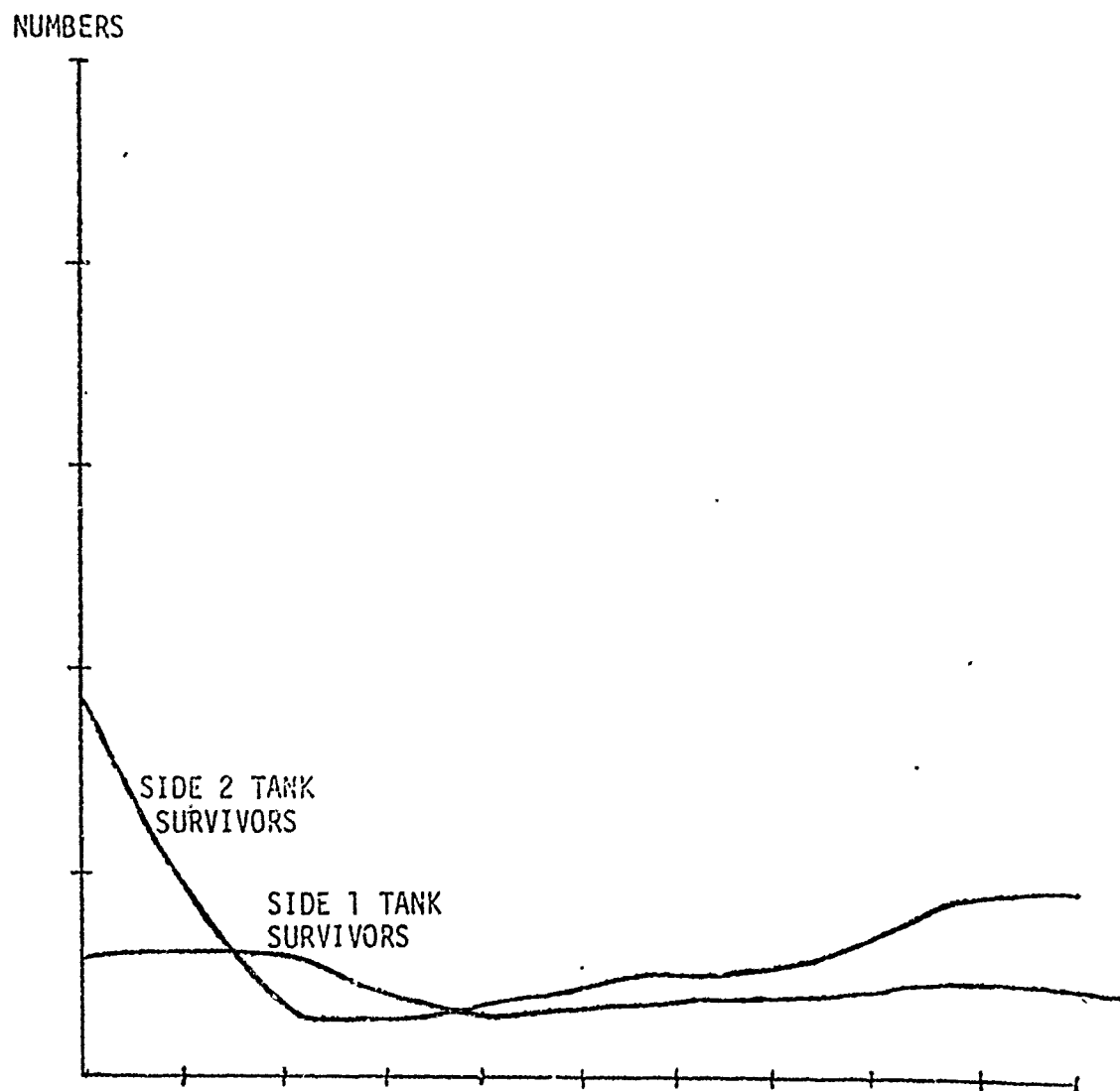
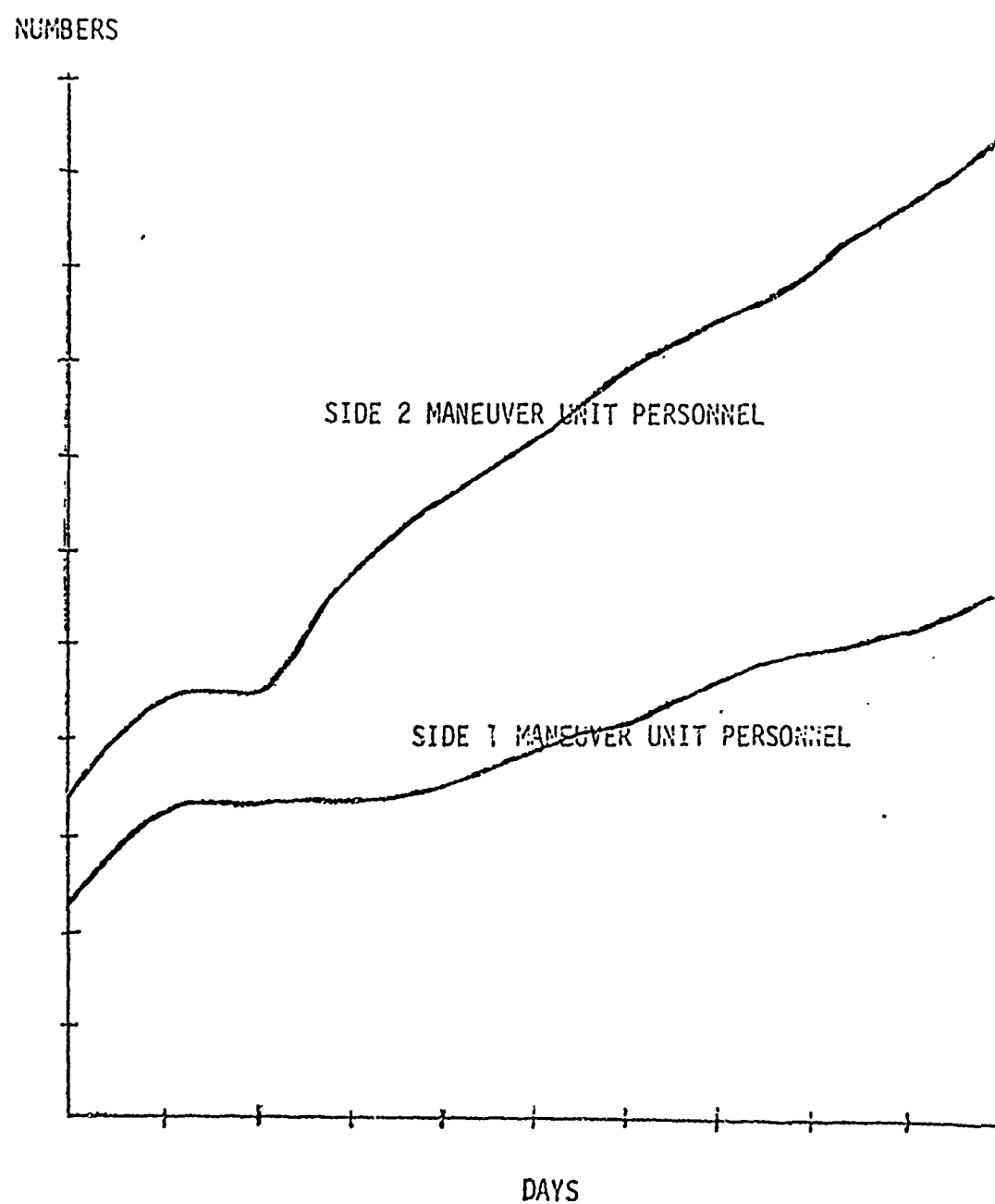


FIGURE 4.5: A HISTORY OF MANEUVER UNIT PERSONNEL  
SURVIVORS IN A HYPOTHETICAL CAMPAIGN



attrited, the resupply, mobilization, recovery, and repair of armor (as input in this run) for the strong-armor force again causes a significant change in the combat conditions.

#### 4.3.3 Sensitivity and Trade-Off Analyses Possible

Because no direct human gaming is required and the model runs quickly, large numbers of runs can be made in the analysis of procurement, deployment, force design, or tactical questions. One of the kinds of output that can be used in procurement and force design is illustrated in figures 4.6 and 4.7 where the effects of variations in the numbers of tactical aircraft and attack helicopters on one specific hypothetical combat of ten days duration are displayed. Comparison of the two graphs does demonstrate one serious difficulty associated with such analyses--the comparative marginal effectiveness of the systems is very sensitive to the performance measure used. This limitation was made even more clear when portions of this analysis were rerun for combat durations of up to 100 days and changes in comparative marginal effectiveness on the same measures of performance were greater than two orders of magnitude. (It may be worth noting that the entire set of runs used in these tradeoff analyses, about 90 runs, were produced in two evenings of computer runs.)

#### 4.3.4 Results are Sensitive to Tactical Rules

Although no extensive analysis of their effects has been carried out to date, the analyses that have been made show that the overall combat results of these models are sensitive to reasonable changes in the input

tactical decision rules in almost every area. This sensitivity reflects the potential sensitivity of campaign outcomes to force missions, strategies, and tactical behavior. It should be noted here that the input rules should represent behavioral assumptions, not doctrinal ones--unless deliberately made. There need be no assumption that the forces in the model can make intelligent, doctrinally correct decisions without hesitation. (Perhaps this observation will answer some of the recent comments that have implied that models cannot play commanders who are not perfect and/or differences in training and ability in decision making processes.)

Overall, it has been the impression of the designers and all those who have used these models that they provide a flexible tool to examine the detailed interactions of tactics and weapon performance in a theater-level campaign and that they provide a structure in which the causal links of the observed output to the input are easily determined for analyses.

More detailed information on the models discussed here may be found in

FIGURE 4.6: ISO-EFFECTIVENESS CONTOURS AS INITIAL  
AIRCRAFT AND HELICOPTERS ARE ALLOWED TO VARY

THE EFFECTIVENESS MEASURE FOR THIS GRAPH IS THE NUMBER OF RED TANK  
LOSSES BY DAY 10.

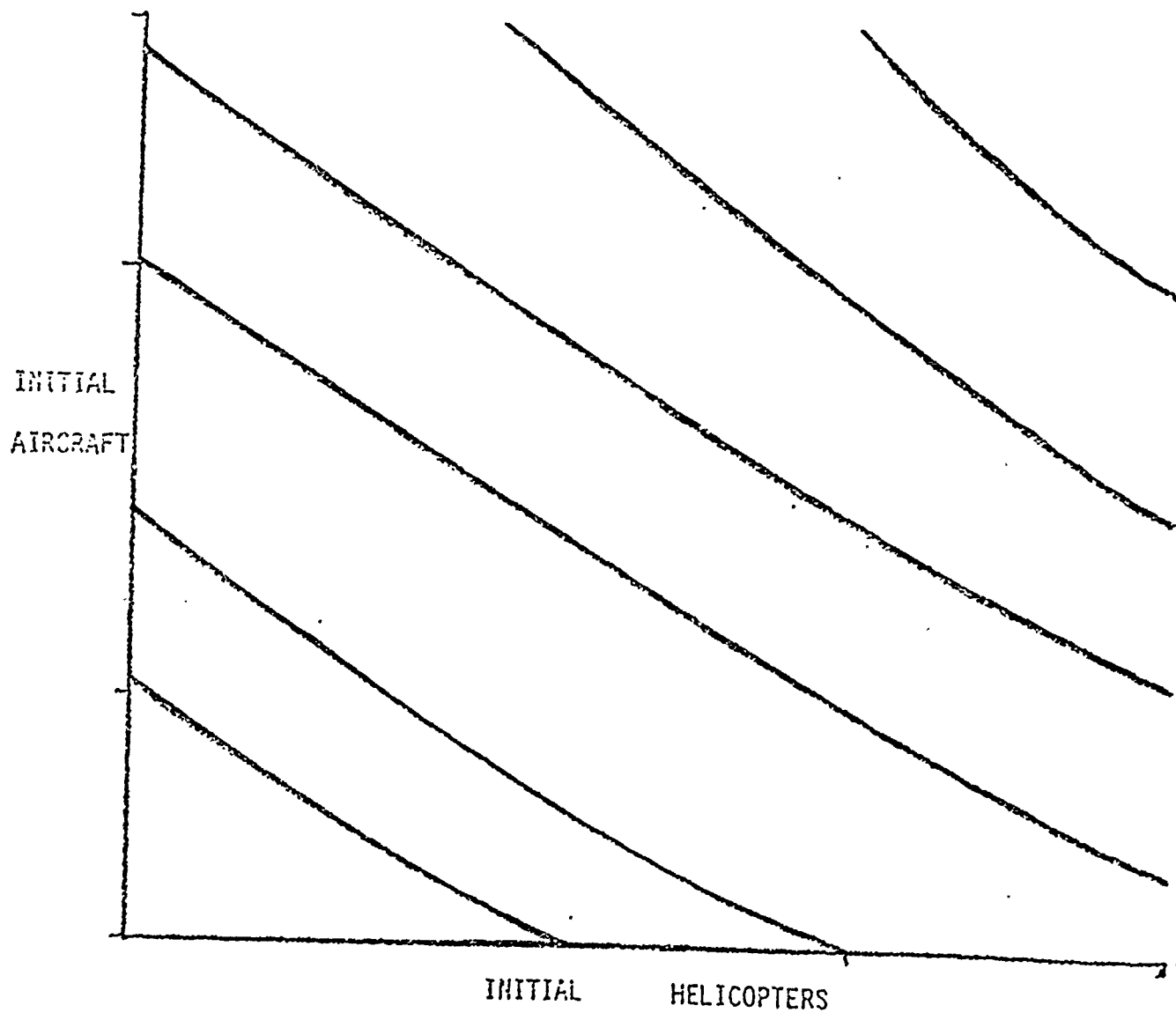
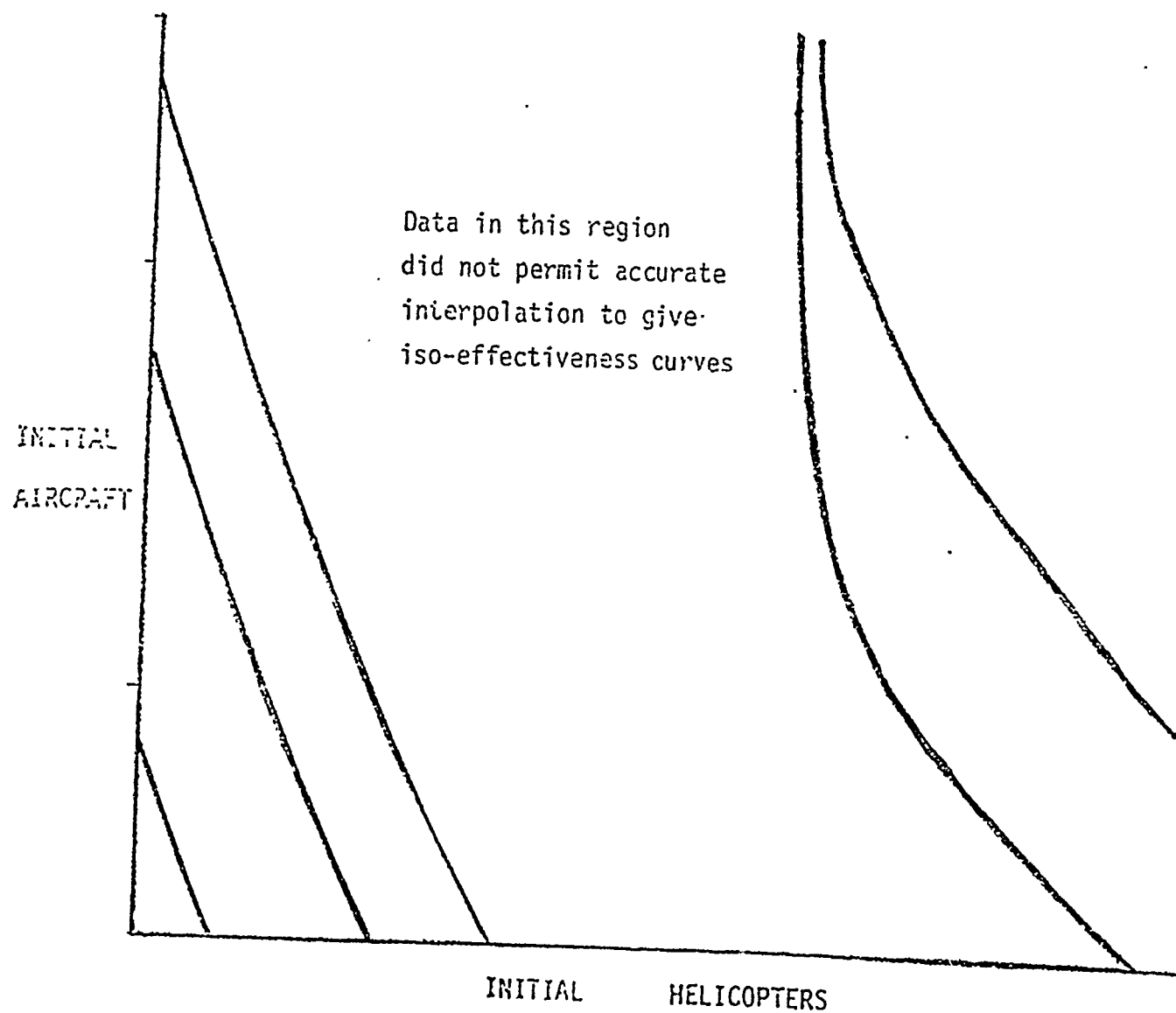


FIGURE 4.7: ISO-EFFECTIVENESS CONTOURS AS INITIAL  
AIRCRAFT AND HELICOPTERS ARE ALLOWED TO VARY

THE EFFECTIVENESS MEASURE FOR THIS GRAPH IS THE MEAN FEBA LOSS BY DAY 10.



## 5.0 BIBLIOGRAPHY

Barfoot, 1969

*The Lanchester Attrition-Rate Coefficient: Some Comments on Seth Bonder's Paper and a Suggested Alternate Method*, Operations Research, Vol. 17, pp. 888-894, 1969.

Bonder and Farrell, 1970

*Development of Analytical Models of Battalion Task Force Activities*, SRL, 1957, FR 70-1, AD 714-617, Systems Research Laboratory, Department of Industrial Engineering, University of Michigan, Ann Arbor, Michigan, 1970.

Lanchester, 1916

*Aircraft in Warfare: The Dawn of the Fourth Arm*, Constable and Company, Ltd., London, 1916.

Parzen, 1962

*Stochastic Processes*, Holden-Day, Inc., San Francisco, 1962.

Vector Research, Inc., 1973

*VECTOR-0, The BATTLE Model Prototype*, Vols. I-V, Weapon Systems Evaluation Group (WSEG), Report No. 222, Arlington, Virginia, 1973.

Vector Research, Inc., 1974

*DIVOPS: A Division-Level Combined Arms Engagement Model*, Report No. ARAFCAS-1 TR-74, Vols. I-II, Ann Arbor, MI, 1974

Vector Research, Inc., 1974

*VECTOR-1, A Theater Battle Model: Volume I, A User's Guide*, Weapon Systems Evaluation Group (WSEG), Report No. 251, Arlington, Virginia, 1974.

Weiss, 1957

"Lanchester-Type Models of Warfare," *Proceeding of the First International Conference on Operational Research*, Wiley, New York, 1957, pp. 82-98.



APPENDIX 5

THE CONTRIBUTIONS OF GAMING AND GAME THEORY  
IN THE STUDY OF FIRE SUPPORT PROBLEMS

THE CONTRIBUTIONS OF GAMING AND GAME THEORY  
IN THE STUDY OF FIRE SUPPORT PROBLEMS  
by Martin Shubik

1. Introduction

There are five topics which this paper will touch on. They cannot be addressed in depth in a paper of this length. However, it is desirable that they be dealt with together. The topics are (a) game theory, (b) gaming, (c) comments on modeling in general, (d) the research and implementation process, and (e) the relevance of the first four topics to the study of fire support.

The first two topics deal with specific methodologies; the third with some general methodological problems, and the fourth with the contextual framework of research and development activity.

2. Game Theory

Game theory is part of a large body of theory dealing with decision-making. It provides a language for the description of conscious goal-oriented, decision-making processes involving more than one individual. It provides a methodology to make amenable to analysis subtle concepts such as the state of information, choice, move, strategy outcome and payoff.

Game theory may be regarded solely as a branch of mathematics which can be studied as such with no need to relate it to behavioral problems, to games or to other applications. However, game theoretic reasoning and analyses are of considerable use in constructing and analyzing games and gaming exercises.

The two major distinctions which must be made by the modeler in utilizing the methodology of the theory of games are the description of structure or the "rules of the game" and the description of behavior or solutions to a game.

Structure

There are three major formal descriptions of a game, each aimed at a different level of aggregation and investigation. They are:

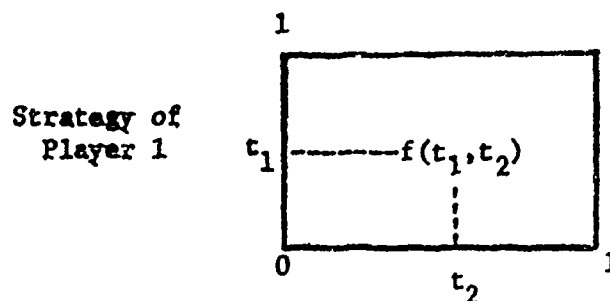
- (1) the strategic or normal form
- (2) the extensive form , and
- (3) the cooperative form of a game (Reference 1)

The first is for systems in which the details of information conditions and order of moves is not of immediate importance. The second representation is designed to stress the details of information. The third representation provides the most aggregated form of a game. Even the details of strategic choice are suppressed. The major concern is with the potential gain from cooperation among the players. This type of game structure is of relevance to diplomatic-military situations such as alliance formation. However, potentially cooperative games with three or more players are not particularly relevant to tactical studies or even to many strategic studies which are best considered as two-sided.

Games in strategic form are most usually presented in a payoff matrix structure. Those most relevant to military models are two-sided games and are represented in finite strategy form by payoff matrices or in a continuous strategy form by games on the unit square. Two examples are shown in Figures 1a and b.

		Strategy of Player 2	
		1	2
Strategy of Player 1	1	15, 5	-1, -1
	2	-3, -2	4, 14

a



Strategy of  
Player 2

b

Figure 1

In Figure 1a, each player has two strategies and the game portrayed is a nonconstant sum game. In each cell of the matrix the payoff to the first, then the payoff to the second player are noted. There is a key distinction in two-sided game theory models between zero-sum games or games of pure opposition, and non-constant sum games. In a nonconstant sum game, the players are not pure opponents; their interests may be positively correlated up to some degree.

Figure 1b portrays a game on the unit square. A player  $i$  has to select an act at some time  $t_i$  where  $0 \leq t_i \leq 1$ . The payoff to player 1 is  $f(t_1, t_2)$  and to player 2 is  $-f(t_1, t_2)$ . An example of such a game is a duel between approaching forces.

#### Behavior

The solution primarily used to investigate two-person, zero-sum games is the well-known maxmin solution (Reference 2). This is a normative solution based upon the proposition that in a game of pure opposition it is possible to define an optimal course of behavior for each side.

This suggested solution is of considerable applied value in the study of weapons evaluation, duels, pursuit and search problems. It and game theory methodology in general depend upon the assumption that there are no coding or computational problems in recognizing and processing information. When limited perceptions and data processing ability are assumed, the game theory analysis must be modified.

When the game portrayal of the conflict situation is best modeled as a two-person nonconstant sum game, then there is no longer a simple single normative theory of behavior which enables us to recommend an optimal policy. Among the solution concepts which can be considered are:

- (a) the noncooperative equilibrium
- (b)  $\max \min (P_1 - P_2)$ ,
- (c)  $\max \min P_1$  and  $\max \min P_2$  (Reference 3)

and there are several others including cooperative and quasi-cooperative solutions.

### Direct Military Applications

There is now a considerable literature on military game theory as is indicated by the book of Dresher (Reference 4), the work of Berkowitz and Dresher (Reference 5), Isaacs (Reference 6) and many others (Reference 7). The models deal with direct weapon duels, or with tactical air fire support, or search and evasion problems. A nonzero-sum game example is provided by Dalkey (Reference 8).

Undoubtedly, most of the game theory models are overly simplified and highly abstract. Nevertheless, they are relatively cheap and where they give paradoxical or strange results, these results may well lead to a clearer identification of what factors have been left out of the model.

### Indirect Military Applications

Although it is my belief that the direct application of game theory methodology is of value to the study of many military problems, probably the more valuable and general use of game theory is in training for strategic thinking and model building of conflict situations. A basic elementary course in the concepts and techniques of game theory for those members of the armed forces concerned with building or sponsoring combat situations could help to avoid many of the problems encountered in the portraying of both environment and behavior.

### 3. Gaming

A Gaming exercise employs human beings acting as decision makers in their own or simulated roles in an environment which in general is simulated. The discipline of gaming deals with the construction, organization, running and analysis of games.

Military gaming in general may be used for operational, teaching, training, experimental or organizational problems. The details of these usages have been discussed elsewhere. In particular, if used with care, gaming can serve to provide an important empirical and organizational check for game theoretic and other analytical models and for computer simulations. Gaming employs a variety of techniques which stretch from the heavily analytic to the unashamedly

behavioral. Two useful categorizations are into environment rich or environment poor games and into free form or rigid games. There is a clear correlation between environment rich and free form games and environment poor and rigid games.

When a game is played as a rigid rule game, at least in theory one could immediately construct a simulation or a well defined analytical game theory model. When a game has free form components, one form of control is exchanged for another. Explicitness of rules are given up in exchange, hopefully for interactive feedbacks among experts. The gaming exercise no longer is purely model dependent, but it becomes part of the model building process.

In military research, possibly the two ends of the range of models can be characterized by the large fully computerized simulation "untouched by human hands", at one end of the range with the highly free form political-military or diplomatic-military exercise at the other end of the range. To the user these two extremes may represent the choice of a scientific mystical black box on the one hand to the choice of a behavioral Pandora's box on the other hand.

A sensitive use of gaming in tandem with game theory and simulation serves as a device to make people consider what is in the black box and in the other direction the game theorists and simulators can by logic and analysis limit the contents of the free form gaming Pandora's box.

As an example of the interaction between game theory and the work that is done in some form of gaming, Table 1 contrasts the game theory (and other rigid rule) analytical model builder's assumptions with behavioral theories.

TABLE 1

<u>Games Theory</u>	or	<u>Behavioral Theories</u>
Rules of the game		Customs and heuristics
External symmetry		Personal detail
No social conditioning		Socialization assumed
No role playing		Role playing
Fixed well defined payoffs		Difficult to define, may change
Perfect human intelligence		Limited human intelligence
No learning		Learning
No coding problems		Coding problems
Primarily static		Primarily dynamic

Direct Military Applications

One cannot make a useful general comment on the success of military gaming in all of its many different uses. Large scale man-machine gaming in real time or slower than real time can be enormously expensive.

Elaborate tactical games may depend delicately upon such details as accurate terrain description and a myriad of parameters which need to be empirically evaluated. Unfortunately, there is no indication that it is possible to evoke some magical law of large numbers to tell us that a host of minor errors will cancel out. It takes an act of faith and a knowledge of the details of one's business to risk claiming that a tactical model is robust enough against variations in the detail.

The justification of gaming usage must be ad hoc. In general, however, it is possible to state that the fewer the questions that are being asked, and the more the questions are well defined, the better are the chances for success with a relatively rigid game. The vaguer the questions, the more important it is to allow for free form components and to stress interaction among experts, builders, users and sponsors.

### Indirect Military Application

The comparing of gaming, game theory and simulation approaches to the same problems teaches a great amount of useful skepticism. Gaming constructively used keeps the simulations and game theory models more honest than otherwise.

The teaching of the methodology of gaming with a stress upon relevant model building and the analysis of play rather than upon playing is an approach which needs to be developed.

#### 4. Some Comments on Modeling

Model building, simulation construction and gaming are, to a great extent, art forms. There is no cut and dried way of learning how to do them well. Furthermore, the task of building MSGs (models, simulations or games) to answer specific questions calls for a considerable amount of hand tailoring. In most instances, the success of the investigation will depend upon the specifics of the model.

The problem faced in furthering the state of the art is to develop an inventory of different techniques and methods which may be general but which provide the means to build highly specific models. General purpose games and simulations, are in general, no purpose games and simulations. (Editor's emphasis).

Although there may be a considerable component of art in model building, there is by now a body of knowledge which needs to be but has not yet been adequately pulled together. The time is at hand for a bringing together of our knowledge of model building and testing. A listing of some of the important items about which we already know a great deal is given in Table 2. It is by no means meant to be complete, but a discussion of even these is hard to find in a single location.



TABLE 2

Data validation	(a) scientific problems (b) organizational problems
Relevance versus realism	
Symmetry	
Sensitivity analysis	(a) conceptual (b) functional forms (c) parameters (d) excluded variables
Aggregation	
External symmetry	
Built in bias and "theology" control	

5. The Purpose and Process

The production of MSGs takes place within a more general decision-making process. Unless we wish to use as our criterion "Validation is a happy customer" and "Replicated validation is a new contract" we must ask a series of process and organizational questions. They include (a) who sponsored the work? (b) did he know what he wanted? (c) who is doing the work? (d) who will use the results? (e) would the sponsor recognize an answer to his question if he had one?

In general, I suspect that most contractors and MSG experts are competent, honest and concerned. Yet the builders of MSGs may easily face a coordination problem with the sponsor that makes it difficult to work with complete openness. It is well within the range of possibility and DoD personnel policy that General A has the belief that a study in a broad, not completely defined area needs to be done, he delegates Colonel B who lets a contract to Consultant Group C who help the Colonel define the study. By the time the study is finished, Colonel B has been replaced by Colonel D, who will have to brief General E, who has replaced General A. The consulting group may find that it, rather than the sponsor, must provide continuity in perception of the motivation for the study. This should not be its job.

An important organizational lesson that is easily learned from game theory is that even if all individuals in different points of an organization are rational and efficient in their own terms, the overall performance of the organization need not be rational or even reasonable. In terms of studies such as detailed fire support simulations, overall organizational rationality must force us to ask how do they fit in, in importance, for the overall organization.

The prime purpose of this paper was to indicate the potential of game theory and gaming methodologies to a specific class of fire support problems. These problems are undoubtedly important in weapons and weapons systems evaluation. They have many technical subtleties involving terrain description measures of effectiveness of weapons, and so forth. Those not intimately aware of the difficulties can only tangentially comment on how these factors influence the potential overall use of such studies. However, almost as an aside, I would like to suggest that the study of the implementation process from the inception of a study, to how it finally influences a weapons mix and force structure problem, is of great importance.

In a different publication, we deal with problems concerning the implementation process in detail (Reference 10); before leaving this topic here, a simple question is suggested which can be appropriately modified and applied to any operational simulation or gaming exercise:

Suppose that in early 1972 a group running a war game had come up with the conclusion that it was possible that the Arabs would be willing to risk raising the price of crude oil to \$13 a barrel. Who would have listened? What actions would have been taken?

This question can be asked of the work in fire support. The answers may well vary with the location of the work, the involvement of the sponsor, and whether the work was in-house or performed by an outside contractor.

The answer to the question on the oil prices "that nothing would be done", is not necessarily bad. Unless the degree of belief is sufficiently high and the results of the study reach sufficiently high levels, there is not enough time to follow up on many future scenarios. The combinations of future states are too large to enable any organization even with the largest of computers to carry out contingent planning in any depth for more than a few broad alternatives.

Actually, several oil experts and economists were aware of what would happen if the Arabs raised the price of oil. This was of sufficient importance that contingency planning would have been highly worth while, but nothing was done.

#### 6. Conclusions

Weapons systems evaluation in general, and fire support problems in particular, pose a considerable number of specific, detailed tactical problems which can only be treated effectively by ad hoc models.

The various methodologies such as game theory, gaming, simulation, dynamic programming, computer language development, human factors analysis, etc. can be used constructively to help to clarify questions and construct better relevant abstract models.

Basic research on game theory, and gaming on weapons systems evaluation appears to be slight. Yet relative to the costs of constructing large specific models such work is relatively cheap.

It is unreasonable to expect that all armed forces personnel at the level of major and above should be experts in any particular methodology; however, it appears to be both possible and desirable to provide education in the uses of model building and how various methodologies play their role. If this is done, the number of incidents of "computer bites man" can be cut down and the quality of models be considerably improved as the understanding of more individuals in the decision process enables them to see more clearly both the value and limitations of simulations.

At the more specifically applied level, it is suggested that game theory models of tactical warfare are cheap and useful as preliminary models to examine measures of effectiveness and to begin to explore a few major variables. Man-machine gaming can serve to bridge the gap between overly simplified game theory models and large black box simulators. They also serve as a means of challenging the assumptions and data hidden in the black box.

Since 1970, there has been an increasing tendency by DoD to abandon man-machine gaming in favor of pure simulations. This trend could be extremely dangerous if the cross-checking uses of man-machine gaming and simple analytical models are not applied to the simulations.

## REFERENCES

1. Shubik, M., Games for Society, Business and War, Elsevier, Amsterdam, 1975, Chapters 2-6.
  2. Von Neuman, J., and Morgenstern, O., The Theory of Games and Economic Behavior, Princeton University Press: Princeton, N.J., 1944.
  3. Shubik, M., op cit., Chapter 5.
  4. Dresher, M.V., Games of Strategy: Theory and Applications, Prentice Hall: Englewood Cliffs, N.J., 1961.
  5. Berkowitz, L., and Dresher, M., "A Game Theory Analysis of Tactical Air War", Operations Research, 7, 1959, 599-620.
  6. Isaac, R., Differential Games: A Mathematical Theory with Applications to Warfare and Pursuit Control and Optimization, John Wiley, New York, 1965.
  7. Fox, M. and G. Kimeldorf (1968) "Strategies and Values in Noisy Duels", Proceedings of the 1968 U.S. Army Operations Research Symposium, 27-34.
- Fulkerson, D.R. and S.M. Johnson (1957) "A Tactical Air Game" JORSA, 5, 704-712.
- Krasovskii, N.N. (1970) "On a Problem of Pursuit", Soviet Math., 11, 343-346. (Dokl. Akad. Nauk SSSR, 191, No.2, 270-272, 1970)
8. Balkey, N.C., "Solvable Nuclear War Models", Management Science, 11, 9, 1965, 783-791.
  9. Shubik, M., op. cit., Chapters 1 and 9.
  10. Brewer, G. and Shubik, M., War, Games and the Computer, Rand, forthcoming, 1976.

APPENDIX 6  
MODELING AND MARKOV PROCESSES

## MODELING AND MARKOV PROCESSES

by Matthew J. Sobel  
Yale University

### 1. OBJECTIVES

This article discusses the modeling and optimization of dynamic processes. Dynamic - or time dependent - processes are embedded in most facets of the utilization of supporting arms in an amphibious assault. The procurement process and the provision of fire support present opportunities for influencing the manner in which these dynamic processes unfold. This realization is an incentive to model the dynamics of fire support.

### MODES OF ANALYSIS

Recent fire support models have been simulated rather than subjected to alternative modes of analysis. This article describes some prominent alternatives to simulation and, it is observed, they may be both applicable to fire support analyses and used in conjunction with simulation in a hybrid fashion.

### OUTLINE

The article begins with a discussion of modeling issues that transcend model type. Then a family of dynamic models is described. To begin with, this family, dynamic network models, is assumed to be free of uncertainty and one-sided; the issues discussed include model size, optimization, computational feasibility, and compatibility with simulation. Then the more realistic element of uncertainty is introduced and the corresponding family of probabilistic dynamic models is described. Again, optimization is introduced and the issues of model robustness and computational feasibility are addressed. The discussion of dynamic probability models is followed by a brief consideration of two-sidedness, i.e., sequential games and their possible applicability to fire support, and of ordinal dynamic processes to accommodate disparate Measures of Effectiveness (MOE's). The article ends with a short list of research and development recommendations.

## 2. MODELING

### MODEL AS HYPOTHESIS

Elementary scientific principles apply to dynamic processes in general and to fire support in particular. A formal model is at first a tentative statement, regardless of its origin in words, a computer program, or mathematics. The generally accepted mode of testing a model's validity is to compare its predictions with observed circumstances. Major discrepancies between observed phenomena and model predictions should induce us to discard or modify the model. A model comes to be regarded less as an hypothesis and more as a useful characterization of reality when its predictions are found to be consistent with empirical reality under widely varying conditions. In the context of fire support, the outcomes of past amphibious landings should have been compared with predictions of the outcomes using fire support models. Such a comparison presupposes battlefield collection of appropriate data. Failing such data and comparisons, it is difficult to identify the levels of detail at which current models should be constructed. To paraphrase Professor New's comment in PGRG's Task 2 Phase 1 ("Fire Support Requirements Methodology Study"), "power and elegance of models and techniques can in no way compensate for lack of understanding or ill-prepared input data."

### DESCRIPTION AND PRESCRIPTION

Numerous dichotomies apply to fire support modeling. The aforementioned advocacy of basic scientific principles bears on relevance vs. realism. Another dichotomy concerns the role of optimization in fire support models. If our objective is to identify desirable mixes of supporting arms and to discover how they ought to be used then our models should optimize the tactics used during an amphibious assault. However, if our objective is to identify mixes of supporting arms that would be effective with existing doctrine then the tactics in our models should describe behavior that would be currently observed. We ought then to consider various models of behavior and optimization models comprise only a part of this collection.



## VALIDATION

Deterministic and stochastic models have already been compared. In principle, it is clear how to validate a deterministic model, i.e., to compare its predictions with empirical observations. The validation process - or "reproducibility" - of stochastic models is less straight-forward. We can hope, at most, to achieve statistical reproducibility, i.e., matching of sample distributions with predicted probability distributions. The dearth of comparisons of fire support models with data, therefore, is in stark contrast with the apparently widely held belief that the efficiency of supporting arms in a given engagement depends on stochastic elements. Some examples of such elements are the weather, the tactics of the opposing force, and the readiness of the opposing force.

This section has posted caveats concerning the validation of models, the optimization of models, and the introduction of stochastic elements in models. These issues will not be mentioned in the sequel although they are implicit in the remainder of the article.

### 3. DETERMINISTIC DYNAMIC MODELS

#### MATHEMATICAL PROGRAMMING

It is convenient to begin a discussion of dynamic models by pretending that the immediate consequences of any decision would be known exactly, in advance. In spite of the effectiveness of intelligence operations, in practice, this state of affairs is never completely realized. However, the idealization simplifies the exposition and the principal issues of modeling and optimization are unchanged by the introduction of uncertainty. Probabilistic - or stochastic - dynamic models will be discussed in 4 for which this section establishes a foundation.

Perhaps our earliest exposure to dynamic models and their optimization occurs in linear and nonlinear programs. Mathematical programs provide a static framework that can be applied to dynamic models. For example, consider the problem of choosing numbers  $x$  and  $y$  that satisfy the constraints :

$$\begin{aligned}x &\geq 0, \quad y \geq 0 \\ y &= (1 - x)(1 + i)\end{aligned}$$

and the criterion

$$\text{maximize } b(x) + b(y)/(1 + i).$$

The number  $i$  is assumed to be nonnegative and  $b(\cdot)$  is a real-valued function on the domain  $(0, 1+i)$ . This static nonlinear programming problem could arise from the dynamic toy problem of partitioning a weapons development budget amount  $1$  between two periods, each being several years long. Quantity  $x$  is budgeted for the first period and, it is projected, the remaining amount available,  $1 - x$ , will have inflated to  $(1 - x)(1 + i)$  by the second period (think of  $i$  as the fractional rate of "interest"). The net benefits of  $x$  are summarized by  $b(x)$  and the discounted net benefits of  $y$  are summarized by  $b(y)/(1 + i)$ .

In a similar fashion, the magnificent but static apparatus of mathematical programming can be applied to optimize less trivial dynamic models. However, the direct application of mathematical programming sometimes masks the dynamic structure of problems. Often it is preferable to exploit the dynamic

structure to obtain computational efficiencies and qualitative insights. In the toy problem above, for example, before embarking on a numerical analysis we should inspect the shape of the benefit function  $b(\cdot)$ . If  $b(\cdot)$  exhibits increasing returns to scale, i.e., if  $b(\cdot)$  is convex (and increasing, then it is suboptimal to forego some of the expenditure until the second period. An optimal solution is given by  $x = 1$  and  $y = 0$  so the entire budget should be depleted during the first period.

Useful dynamic models, particularly models of fire support processes, are unlikely to be as simple as the two-variable example above. Therefore, formalisms are needed to define more complex models. One of simulation's principal virtues is the relative ease with which simultaneous transactions can be described. Suppose transactions  $a$  and  $b$  have starting times  $s_a$  and  $s_b$  and completion times  $c_a$  and  $c_b$ . A simulation can permit  $s_a < s_b < c_b < c_a$  without major difficulty. Simultaneity of this kind is disagreeable in the dynamic network models that are about to be discussed. However, we shall see that some simultaneity can be tolerated without resort to simulation.

#### DYNAMIC DECISION NETWORKS

Dynamic decision networks comprise a useful formalism for many complex dynamic models. Such a network is a collection of nodes connected by directed arcs. Each node in the dynamic model is associated with two data: (a) a point in time or a chronological stage in the unrolling of the dynamic process; and (b) a military state of affairs. The state of affairs should summarize the current situation as it pertains to the decisions currently faced. The state of affairs - or "state" - might include intelligence on size and disposition of enemy forces, deployment of USMC supporting arms, and logistics considerations. The "states" in VECTOR-1, for example, are not unlike the network nodes currently being discussed. The nodes in BALFRAM, however, denote geographical locations and are quite different from decision network states.

An enormous number of network nodes may be needed at each chronological stage. The feasibility of using network models is limited primarily by the number of nodes at each stage and by the number of stages. Consider, for example, a stage at which the immediate beach area has been secured. Suppose

that the state depends on the status of intelligence, deployment of supporting arms, and logistics considerations. If intelligence status can be placed in one of 20 categories, arms deployment in one of 10 categories, and logistics status in one of 5 categories, then there are  $20 \times 10 \times 5 = 1,000$  states, or nodes, at the stage being discussed. Of course, different stages in the amphibious assault entail different numbers of states but, for the sake of illustration, suppose there are 20 stages and that each stage has 1,000 states. Then the entire network contains 20,000 nodes! Such a network is impressively large but not prohibitively huge from a computational point of view.

### ARCS

Several arcs may emanate from a network node. These arcs correspond to the alternative decisions that are available in a particular state of affairs at a given chronological stage. For example, different matchings of targets to supporting arms elements may be considered. The number of alternatives, therefore arcs, is relatively low if existing (or proposed) doctrine is assumed. The number of arcs increases as commitments to doctrinaire decision-making are relaxed.

If uncertainty is absent then it is possible to predict accurately the immediate consequences of taking a particular action while in a given state at a specific stage. This assertion is equivalent to knowing which state would be occupied at the subsequent stage. Therefore, an arc connects the node at which the corresponding decision could be taken with the node which describes the new state of affairs at the subsequent stage.

The preceding network example had 20 stages and 1,000 states, i.e., nodes, at each stage. If 5 alternative decisions were to be available at most nodes then the network would contain approximately 100,000 arcs connecting 20,000 nodes.

### SIMULATION

Many simulations consist of traversing the network from a distinguished starting point, or "origin," to a subset of specified nodes, termed "sinks". The simulation program, in effect, describes for each node a switch whose

settings are the actions described above. The program or input data specify the action to which each node's switch should be set. A simulation "run" evaluates the characteristics of the route through the network that results from the input parameters and switch settings. Numerous expensive runs are sometimes made to compare the effectiveness of different combinations of switch settings.

Network optimization algorithms, by comparison, have each switch's setting as a variable and they compute an optimal combination of settings for all the switches in the network. "Optimality" here has a severely limited scope. The measure of effectiveness (or weighted average of measures of effectiveness) must be represented as a sum of numbers attached to the arcs actually traversed in a route from the source to a sink. Every arc in the network is assumed to be labelled with such a number and every route from source to a sink is labelled with the sum of the numbers associated with the arcs in the route. A route is optimal if its sum is at least as great as the sum associated with any other route.

A single "run" of the optimization algorithm encompasses the routes generated by all possible combinations of switch settings. In the preceding network with 1,000 nodes at each stage, and 20 stages, and 5 alternative decisions at each node, for example, there are  $1000 \times 5^{19}$ , different routes (no arcs emanate from the nodes in the last stage).

A major virtue of network optimization algorithms is their implicit comparison, for a single MOE, of a staggeringly large number of alternative routes. The cost of this virtue is that each node in the network must be considered explicitly. The optimal solution includes a contingency plan that specifies an action at each node that would be optimal if that node were visited in a route through the network. However, many nodes would be avoided by every sensible route and, certainly, an optimal route. Therefore, the optimization algorithm obliges us to accept a specification for a large number of nodes that are absurd. Often these absurd nodes cannot all be pruned from the network because their absurdity is recognized only from a post-optimality analysis.

Simulations usually avoid the price of having to visit all nodes including those that are absurd. Because the switch settings are input data or in the program, only one node is visited at each stage so the number of nodes visited is equal to the number of stages. Therefore, vastly greater networks can be simulated than can be optimized. However, the cost of simulation that one saves with optimization is that only a miniscule fraction of the possible routes can be evaluated.

Amphibious landings are sufficiently complex and lengthy operations that their corresponding network models are likely to be large by any standard. It seems unlikely that it will be possible to optimize such networks in the foreseeable future and, therefore, simulation of some kind is inevitable. However, major segments of the network are probably susceptible to optimization; some fire support examples will be given in the sections on stochastic networks. Ad hoc methods have been used for two decades to embed optimization in simulation but the art of such hybrid procedures is primitive and the science is nonexistent.

R&D Recommendation 1. Develop the art and foster the science of simulation-optimization hybrid procedures.

#### OPTIMIZATION

The time-staged network optimization problems above are, in fact, discretized dynamic programming problems. It is widely supposed that the "curse of dimensionality" prevents the optimization of problems whose states are composites of several individual variables. However, the "curse" is partly myth because the optimization of time-staged networks can be accomplished with extraordinarily fast algorithms. Network optimization problems are specially structured (ref. 1) linear programs and, more than a decade ago, it was not uncommon to encounter problems having 10,000 constraints and 50,000 variables. In the network context, we have constraint for each node and a variable for each arc.

The time-staged structure of our networks makes it possible today to optimize networks of olympian size. However, no software package (to my knowledge) is specifically adapted to time-staged networks. Therefore each user would have to tailor the input and output formats and the data editing



#### 4. STOCHASTIC DYNAMIC MODELS

The use of stochastic models entails the collection of much more data than the use of deterministic models. In the first case one estimates a probability distribution's dependence on parameters; in the second case one estimates "merely" a numerically valued function of the parameters. However, this comparison is not a sufficient reason to use a deterministic rather than a stochastic model. In each context, say fire support, the issue is the significance of the stochastic elements in connecting effects to their causes. For example, suppose the outcome of an engagement can turn on reaction times when intelligence is less than perfect and suppose intelligence gaps occur sporadically. Then too little value may be attached to short-run flexibility of supporting arms mixes if engagements have been described with a deterministic model. In other words, the issue is whether or not the explicit inclusion of stochastic elements is likely to yield significantly better decisions.

##### STOCHASTIC PROGRAMMING

Stochastic programming is the part of mathematical programming which addresses the effects of uncertainty in the objective function or constraints. However, the existing theory and algorithms seem ill-suited to the analysis of very large problems. Instead, we shall turn to theory and algorithms from dynamic programming - or Markov decision processes.

##### STOCHASTIC NETWORKS

Nodes in the deterministic networks denoted time-staged alternative states of affairs - or "states". Nodes in stochastic networks play the same roll but offer greater flexibility in the treatment of intelligence. Several "realities" often are consistent with the limited information at hand and, at best, only one's degree of belief in each alternative reality can be specified. "State," therefore, can encompass the posterior distribution used in a Bayesian treatment of information.

Arcs in stochastic networks differ significantly from their deterministic counterparts. The presence of uncertainty makes one unable to predict the exact consequences of a particular action. Therefore, one must distinguish



a transition connecting successive states from an action available at a given state. Different actions may cause the same transition, and any of several possible transitions may result from a single action. In a stochastic network, at each node (state) and for each alternative action at that node, it is necessary to stipulate the probabilities associated with transitions to the various nodes possible at the successive stage in time. The decision trees of "decision analysis" are examples of such networks. An example in the target designation process would result by modifying the queuing model in DAFS/CAS.

It was observed previously that simulation models are particularly well suited to the inclusion of simultaneous events. Stochastic network models can describe such events by augmenting the state with supplementary variables. This technique is standard in applied probability, particularly in queuing theory. However, the flexibility offered by supplementary variables is obtained at the price of a geometric increase in the number of augmented states. It is routinely assumed that such a price is too high and that the only reasonable recourse is to simulation models. The empirical conditions under which the assumption is valid are unknown and undoubtedly simulation is used sometimes when explicit network models or hybrid models would be more appropriate. As was advocated in the first R&D Recommendation, the trade-offs and hybridization possibilities should be explored. For example, is it prudent to embed a target designation stochastic network model in a large fire support simulation?

#### OPTIMIZATION

The optimization issues are similar for deterministic and stochastic networks. Particularly efficient optimization algorithms (Ref 2, 10) are available but corresponding software packages for time-staged networks have not yet been developed. The existence of incommensurate vaguely defined MOE's is a more fundamental problem which will be addressed briefly in section 5.

Network optimization models are particularly well suited to sensitivity analysis. The problems are, fortuitously, linear programs which have well established theory and numerical procedures to analyze their sensitivity (Reference 2, 10).

T. Horrigan has advocated (Reference 3, Appendix D) the exploitation of special structures in simulations. It is generally prudent to exploit special structures wherever they are found and this homily is as true for dynamic optimization as it is for simulation. For example, there is a well-established and growing literature on the optimal operation of queues (Reference 7). Optimization of the target designation process appears closely related to recent and on-going research of this kind. The research elicits justifications for considering only simple and easily implemented policies. Such policies typically comprise a small fraction of the collection of all possible decision rules. Therefore, it is much more efficient to seek the best simple policy than to search for the best among all decision rules.

## 5. OTHER ISSUES

This section briefly describes some recent research results that may be useful in analyzing the effects of (a) two-sidedness and (b) several measures of effectiveness (MOE's).

### TWO-SIDED DYNAMIC MODELS

Two-sided models invest both warring parties with strategic richness. It is easy, in principle, to construct such dynamic models but more difficult to analyze them. Suppose, for the moment, that specific strategies are attributed to Blue and Red. The resulting dynamic process, at its most general, is a stochastic network of the kind described in Reference 4. From any node or state of affairs, a stochastic transition to another state occurs as a joint consequence of the actions of both Blue and Red at that node. The most important impediment to the analysis of such network fire support models is the difficulty of constructing them. Professor Shubik's caveats (Reference 6) in this report are applicable to this task. If such a model were constructed, how should it be analyzed? Two options, described below, are solutions of stochastic games and optimization of pseudo-two-sided models.

A stochastic game is a jointly controlled network as described above. It has the added feature of benefits or costs being incurred by each player during each transition from one state to another. The scope of such sequential game models is exceedingly broad (Reference 5). The solution criteria that have been investigated are saddle-points for zero-sum games and, otherwise, equilibrium points. An equilibrium point is a pair of strategies, for Red and Blue, with the property that each member of the pair is a best rejoinder to the other member. The existing theory has the serious apparent limitation that Red and Blue are assumed to share their awareness of the current state and the structure of the entire network. It is unclear whether this limitation can be removed.

Until recently, it was impractical to compute equilibrium point solutions (or saddle-points in the zero-sum case) to dynamic games. Now it seems that the earlier difficulty resided in the assumption that Red and Blue always could make simultaneous decisions. Instead, if they are assumed to alternate their opportunities to make decisions, then the stochastic game can be solved as a linear programming problem (Ref.9). The application of this result has the virtue of convenience and the danger of ignoring other solution criteria (Ref.6). Nevertheless, the infant art and science of numerically solving large stochastic games should be nurtured.

R & D Recommendation 3. Develop the art and foster the science of numerical solutions of sequential games.

Stochastic games invest the opponent with strategic richness but many simulations deny him any strategy and, instead, stipulate his actions as time passes. A strategy is a contingency plan and an intermediate approach would be to endow the opponent with a contingency plan. Then his actions would be in reaction to our own. He would not choose from among alternative contingency plans, as in a stochastic game. On the other hand, while being a "strategic dummy" he would not also be a "reactive dummy." Such a pseudo-two-sided model is again a stochastic network of the kind discussed in Section 4, so its optimization is straightforward. A move towards a pseudo-two-sided model is the internal generation of target lists as DYN-TACS seems to do.

SEVERAL MEASURES OF EFFECTIVENESS

The optimization discussed thus far concerns a single numerical MOE. Of course, this includes a weighted average or other numerical function of several MOE's. However, it has been argued at this Workshop that disparate MOE's are sometimes simultaneously present. We shall take "disparate" to mean that the preferences of the mythical (nonexistent?) decision-maker perhaps may not be representable by a weighted average of elementary MOE's.

"Utility theory," "decision theory," and "decision analysis" are labels for literatures that explain how one "ought" to make decisions even in the absence of a numerical single MOE, i.e., in the absence of cardinal utility. However, most of this theory concerns static decision problems, whereas fire support problems are dynamic. The essence of a dynamic process is the dependence of its "location" at later stages on the decisions made at earlier stages. Therefore, optimization of dynamic processes invites the preparation of contingency plans. Contingency plans, instead of inflexible time-tables, seem necessary when operations occur in an uncertain environment against a partially unpredictable adversary. The part of utility theory which is not restricted to an assumption of cardinal utility is ostensibly static and seems ill-adapted to planning for contingencies, i.e., dynamic optimization.

Recent research provides some theory and algorithms for dynamic optimization (Ref. 3, 8) in the absence of cardinal utility, (although no such results exist for the previously discussed sequential games). This recent material, labelled "ordinal dynamic programming," begins with the assumption that the "decision-maker" is able to rank the outcomes of the dynamic process. "Outcome" here means the entire time-dependent sequence of states (or statuses or nodes) visited and actions taken, i.e., the history of the engagement reviewed after its conclusion. Ties in the rankings might be widespread. However, it is assumed that the rankings satisfy certain postulates of rationality.

At least two obstacles arise at this point. First, who is the decision-maker and how does one elicit his preferences, i.e., his ranking of alternative outcomes? It may be simpler to elicit a ranking using vector-valued MOE's than to identify the decision-maker (for purposes of fire support analysis). The second obstacle is that a ranking, once obtained, might be fundamentally inconsistent with the rationality postulates in References 3 and 8. If these obstacles can be surmounted, then the algorithms in References 3 and 8 are, in principle, essentially the same as those that pertain to the dynamic networks in Sections 3 and 4.

R & D Recommendation 4. Investigate the applicability of ordinal dynamic programming to fire support analysis with disparate MOE's.

## 6. R & D RECOMMENDATIONS

The four recommendations in the text are restated below:

- \*1. Develop the art and foster the science of simulation-optimization hybrid procedures.
- \*2. Develop software packages for the optimization of dynamic (stochastic) networks.
3. Develop the art and foster the science of numerical solutions of sequential games.
4. Investigate the applicability of ordinal dynamic programming to fire support analysis with disparate MOE's.

It seems to me that recommendations 1 and 2 are interrelated and more essential than 3 and 4. Also, 1 and 2 are probably more expensive than 3 and 4.

## REFERENCES

1. Elmaghraby, S.E., Some Network Models in Management Science, Springer-Verlag, 1970.
2. Hillier, F. S. and G.J. Lieberman, Operations Research, second edition, Holden-Day, San Francisco, 1974.
3. Mitten, L. G., "Preference Order Dynamic Programming," Management Science, Vol. 21, pp. 43-46, 1974.
4. Potomac General Research Group, "Fire Support Requirements Methodology Study: Review of Theories and Techniques Applicable to Marine Fire Support," contract M00027-74-A-0042 (unclassified), June, 1975.
5. Sanghvi, A., M. Shubik, and M. J. Sobel, "Bibliography on Sequential Games," unpublished manuscript, 1974.
6. Shubik, M., "Uses of Game Theory, Gaming, and Model Building in the Study of Fire Support Problems," this report.
7. Sobel, M.J., "Optimal Operation of Queues," in Mathematical Methods in Queuing Theory, A.B. Clarke ed., Springer-Verlag (Lecture Notes in Economics and Mathematical Systems #98), Berlin, 1974, pp. 231-261.
8. Sobel, M.J., "Ordinal Dynamic Programming," Management Science, Vol. 21, pp. 967-975, 1975.
9. Sobel, M.J., "Alternate-Play Stochastic Games," unpublished manuscript, 1975.
10. Wagner, H. M., Principles of Operations Research, second edition, Prentice-Hall, Englewood Cliffs, N.J., 1975.



APPENDIX 7

SIMULATION AS A METHODOLOGY FOR  
FIRE SUPPORT STUDIES

SIMULATION AS A METHODOLOGY FOR  
FIRE SUPPORT STUDIES

by

George S. Fishman  
University of North Carolina  
Chapel Hill  
July 1975

1. Introduction

Although references to simulation models in the Review of Theories and Techniques Applicable to Marine Fire Support [9] indicate a general dissatisfaction with their contribution to progress in understanding the nature of Marine fire support activities, most attendees at this conference would agree that some form of simulation model nevertheless will play a role in future fire support studies. Given this inevitability, one hopes that the problems encountered in the past can be either avoided or at least ameliorated in the future by some thoughtful reflections on their origins and on their possible solutions. In today's presentation my remarks are intended to put the problems that arise in a simulation of fire support in perspective and to offer direction. Hopefully, this perspective and direction will facilitate our discussion here on just what can be done to increase user satisfaction with fire support simulations.

## 2. Features of the Simulation Method

As a tool for studying complex systems, simulation offers many attractions. These include:

1. compression of time
2. expansion of time
3. model detail
4. selection of outputs
5. control of measurement errors
6. control of variation.

A properly constructed simulation model can compress time so that several years of system activity can be simulated in minutes or, in some cases, seconds. This ability enables one to run through a variety of operational designs of interest in a fraction of the time required to try each on the real system.

The ability to expand time also has its benefits. By arranging for statistics of interest to be produced over small intervals of simulated time, one can study the detailed structure of system change that cannot be observed in real time. This figurative time dilation is especially helpful when little data exist on change in the real system.

Model detail is often cited as the most notable feature of computer simulation. Although all modeling involves some abstraction from reality, the ostensible reason for using simulation in the minds

of many analysts is that it allows them to model detail that other methods would have to omit in order to admit a solution. This ability to include detail has occasionally led to a euphoria about what simulation can do. Unfortunately the dark side of the picture is seldom mentioned in advance and inevitably a user who exploits this ability to include detail learns that all is not well at a later stage in his use of simulation. Section 3 discusses the subject of detail with regard to its dark side in detail.

The ability to select output and reports of varying degrees of detail also contribute to the appeal of simulation. However, it should be remembered that the computation of output statistics take time. Therefore, a judicious simulation user devotes prior thought to what the relative importance of different outputs is and to the ways in which he can manipulate a small internal data base to produce many outputs of interest. For example, in a queueing system the identity  $L = \lambda W$  where  $L$  denotes mean queue length;  $\lambda$ , the arrival rate; and  $W$ , the mean waiting time holds under fairly general conditions. Therefore, one need collect data to estimate either  $L$  or  $W$  since the other can be obtained by either division or multiplication by the known arrival rate  $\lambda$ .

Control of measurement errors offers a great comfort to simulation users. Presumably the automatic fashion in which data are collected in a computer simulation together with the fact that machine errors are virtually absent has, until recently, led to complacency about the possibility of error. With the advent of the simulation of

computer systems in which the time between events is of the order of micro-seconds but run lengths are of the order of hours, it has become clear that the accumulated simulation time which is generally computed by adding the times between events is subject to substantial error. Whether or not this is a serious issue for fire support studies depends on the nature of interevent times relative to run length times.

Control of variation is the least appreciated feature of computer simulation. This may be a result of the fact that some knowledge of statistics is necessary to exploit this feature. In particular, application of this control of variation enables one to obtain results with a specified accuracy at lower cost than if one ignored the potential for control. Section 4 offers a number of examples to illustrate how easily this exploitation can be made to work.

### 3. Detail

There exists a general presumption among analysts that if they were just able to make their models conform more closely to the observed behavior, then they would increase chances of having a successful study. Simulation, being a descriptive tool, allows one in theory to make a model as close to resembling reality figuratively as one likes. However, in order to close the gap between model and reality, one has to have a definitive picture of the behavior to be modeled.

Fire support involves a host of microphenomena; they include:

- A. target acquisition
  - 1. detection
  - 2. identification
  - 3. location

B. target engagement

1. priority rules
2. weapons availability
3. weapons selection rules

C. fire support performance

1. target characteristics
2. weapons characteristics
3. measures of effectiveness.

Each calls for detail which hopefully would arise from actual battlefield experience. If one were to judge from the sampling offered in [8], this detailed knowledge is either absent or not used in modeling. For example, take the discussion of target acquisition on p. 3-4. The probability of detecting a target is computed on the assumption that all sensors are "equally involved in the acquisition and perform independently." Although this writer recognizes the difficulty of working without this assumption, its plausibility seems open to question. In addition, the presentation in [8] omits any mention of how the probabilities of locating and identifying a target vary with time, once detection has occurred.

If the knowledge needed to derive a more adequate representation of target detection, location and identification exists, then one has to decide whether its inclusion in the simulation will improve representational accuracy to an extent that makes the extra modeling effort worthwhile. However, this improvement can only be measured after the fact. In particular, inclusion of known

detail in a comprehensive fire support description would have to be preceded by extensive testing of alternative mathematical and logical representations. To do this one needs data.

Every extension of a simulation's detail introduces new parameters. These require estimation which relies on data, whether it be sample observations or expert judgment. Naturally the more detail that is desired, the more data that are required. This poses a dilemma for the analyst. While he may be able to describe a phenomenon conceptually, he may not have the data needed to fit the parameters of the corresponding mathematical representation. If he does have the data, he must then face the issue as to how representative the parameter estimates are when this particular micromodel is used in a variety of alternative settings. That is, parameter values may be a function of the setting in which the model is used and, therefore, an analyst may need several sets of data to estimate the values that parameters assume in different settings.

The third dark issue that more detail induces is increased bookkeeping and computation in a simulation computer program. More detail implies more events or state changes per unit time in the model. From a programming viewpoint this requires additional data structures and logical structures. This requirement adds to the cost of putting the program together. Although it is true that languages such as GPSS and SIMSCRIPT II make these supplements relatively easy to introduce representationally, an analyst is still faced with the problem of fitting his program into the computer on which he plans

to do his work.

If FORTRAN is used for modeling then a serious additional problem arises. Fire support simulation involves relatively intricate time sequencing of many diverse events. Whereas specialized simulation programming languages all contain timing routines that perform this time sequencing automatically, the user of FORTRAN must build his own timing routine. This effort alone can be so cost consuming as to defeat the purpose of using FORTRAN for its computational efficiency. In particular, FORTRAN lacks a list processing capability, a principal feature of all simulation programming languages. For this reason alone one has to question the flexibility and versatility of a fire support model programmed in a language other than a simulation programming language.

The effect of detail on program development represents only one issue in this area. Detail seriously affects program execution also. In addition to creating more data and logical structures, more detail causes more events to occur per unit time in a simulation. This implies that the list of scheduled events on which the timing routine relies for direction is longer. This means that when a new event is to be scheduled the timing routine takes more CPU time to find the correct position for the corresponding event notice in the list of schedule of events.

Unfortunately the current state of development of most simulation languages have contributed to the seriousness of this problem in practice. In order to retain a simplicity in list



structures and processing for general simulation, these languages search, add and delete from these lists using algorithms that in no way exploit the nature of the event list for particular problem settings. Moreover, many simulation users do not recognize that alternative ways exist to process the list of scheduled events as well as other lists that materialize during the course of a simulation.

Appendix D of the Review of Theories and Techniques Applicable to Marine Fire Support [9] recognizes that the generality of simulation programming languages may represent an impediment to computational efficiency in the fire support area. This recognition has led to a proposal there for more tailoring to the needs of this kind of simulation. This idea deserves encouragement. However, one hopes the tailoring will not be restrictive of the resulting simulation programs' use for alternative fire support studies. Using a simulation language to formalize concepts and structures would help to insure this generality.

Few, if any, tailored simulations have been reported in the literature. What has been reported are ways to speed up list processing in general. One suggestion which most experienced simulation users follow, regardless of the problem, is to create a single event notice for two diverse events that always occur simultaneously. Then a subroutine call within the executable code of one of the events enables the other event to be executed. A second suggestion concerns conditionality. Occasionally one event occurs only after another type of event has occurred. However, the second event does not always occur. In this case an event notice for the necessary event is generated in the simulation and within the executable code for this event a test is made to see if execution of the other type of event has to

occur. The effect of these two suggestions is to reduce the number of event notices in the list of scheduled events, thereby reducing the processing time for this list. Unfortunately the very emphasis on events in a language such as SIMSCRIPT encourages a user to overlook the fact that simple suggestions such as these two can considerably shorten execution time.

Recently, other suggestions have appeared in the literature. The papers by Vaucher and Duval [11] and Wyman [12] in the Communications of the ACM relate experience with alternative search procedures aimed at reducing list search time. In GPSS the judicious user of a user chain to shorten the length of the current events chain offers dramatic savings, when properly used [6].

Improved processing of other lists can also induce efficiencies in large scale simulation. For example, suppose that available resources in a fire support simulation are all kept on a single available resource list. Presumably the type of resource is distinguished by a value assigned to its attribute that designates type. Every time a resource is required a search of the resource list occurs. If there are many available resources of many different types the search is time consuming. Alternatively, if one judiciously constructs several lists based on type then the simulation needs only to search the selected shorter list. The price paid for search efficiency is the increased number of list structures defined in the simulation. The exact balance between the cost of having more lists and the saving in search time depends on the particular system under study.

#### 4. Control of Variation

Although control of variation seldom receives serious attention in large scale simulation, it is in this writer's mind one of the most attractive features of the simulation method. Control of variation includes the ability to control the pattern of variation in the streams of random numbers that serve as input to an ongoing simulation. Thoughtful use of this ability enables a user to attain a desired statistical accuracy with less computer time than neglect of the option would require. This benefit can accrue when running replications of an experiment in which all input parameters are the same. It can also occur when comparing runs of an experiment in which at least one of the input parameters assumes different values. An example illustrates the point.

Consider an airline reservation office with  $m$  reservationists. If at least one reservationist is idle when a call occurs the call immediately receives service. If all reservationists are busy the caller listens to a 9 second recorded message excusing the delay. At the end of the message the caller receives service, if a reservationist is available. Otherwise, he is put into a queue with first-come-first-served discipline. Intercall times <sup>follow</sup> an exponential distribution with mean  $1/\lambda$ . Each caller makes a one-way reservation with probability  $1-p$  and a round trip reservation with probability  $p$ . Service times for one way trips are exponential with mean  $1/\omega$ . Round trip service times are Erlang with shape parameter 2 and mean  $2/\omega$ . Times are in minutes.

Consider the case in which  $\lambda = 1$ ,  $\omega = 0.5$ ,  $m = 6$ , and  $p = 0.75$ . Suppose one wishes to estimate mean waiting time to within  $\pm 0.025$  min-

utes or, equivalently,  $\pm 1.5$  seconds. Let  $Y_i$  denote sample mean waiting time on replication  $i$ . Let

$$(1) \quad \bar{Y}_k = k^{-1} \sum_{i=1}^k Y_i \quad s_k^2(Y) = (k-1)^{-1} \sum_{i=1}^k (Y_i - \bar{Y}_k)^2.$$

Suppose we adopt the following design for our experiment: Continue to collect independent replications until [1]

$$s_k^2(Y) \leq k(0.025)^2/t_{k-1}$$

where  $t_{k-1}$  is the .975 significance point of the  $t$  distribution with  $k-1$  degrees of freedom. Then if  $Y_1, \dots, Y_k$  are normally distributed the probability that  $\bar{Y}_k$  is within  $\pm 0.025$  of the true waiting time is approximately<sup>†</sup> 0.95. Table 1 shows the results using independent replications.

This particular simulation was run in SIMSCRIPT II.5 with intercall times generated on stream 1, service times on stream 2 and type of call (one way or two way) on stream 3. In a simulation of a single server queueing system Page [7] has shown that reversing the streams of random numbers for interarrival and service times on a second replication can induce sizable variance reductions. Presumably, low interarrival times and high service times produce high congestion on the first run whereas reversal of streams produces high interarrival times and low service times and, therefore, low activity on a second run. Therefore, average sample output over the two runs should have a smaller variance than in the case of independent replications.

Table 2 presents the results of reversing seeds on streams 1 and 2 on pairs of replications. In order to allow comparison with

<sup>†</sup>See [10] for details.

Table 1.

Sequential Estimation of Mean Waiting Time

 $d = 0.025$  minutes, significance level = 0.05

$k$	$y_k$	$\bar{y}_k$	$s_k^2(Y)$ ( $10^{-4}$ )	$kd^2/t_{k-1}^2$ ( $10^{-4}$ )	No. of Completions
1	0.2243	0.2243			2651
2.	.1705	.1974	14.47	0.08	2497
3.	.1721	.1890	9.37	1.01	2781
4.	.1619	.1822	8.08	2.47	2500
5.	.1583	.1774	7.20	4.06	2629
6.	.2275	.1858	9.94	5.67	2550
7.	.2222	.1910	10.18	7.31	2595
8.	.1576	.1868	10.12	8.94	2422
9.	.2362	.1923	11.56	10.58	2440
→ 10.	.2138	.1944	10.74	12.22	2478
					25543

Table 1 the experiment here was designed to have about half as many completions per run as in Table 1. The results in Table 2 indicate that only 12354 completions were required to obtain the same statistical accuracy as in Table 1, which required 25543. In terms of variance one has

$$(2) \quad s_5^2(X) = 7.55 \times 10^{-4} \quad s_5^2(Z) = 23.34 \times 10^{-4}.$$

Then one way to measure variance reduction is to examine the sample ratio

$$(3) \quad [s_5^2(X) + s_5^2(Z)]/4s_5^2(Y) = 2.03$$

which indicates that seed switching has cut the variance by about one half.

Table 2,  
Sequential Estimation of Mean Waiting Time  
Using Seed Switching  
 $d = 0.025$  minutes, significance level = 0.05

k	$\lambda_k$	$Z_k$	$Y_k = (X_k + Z_k)/2$	$\bar{Y}_k$	$s_k^2(Y)$ ( $10^{-4}$ )	$kd^2/t_{k-1}^2$ ( $10^{-4}$ )	No. of Completions
1	0.1649	.1699	0.1674	0.1674			1262 + 1335
2	.2240	.2338	.1739	.1707	2.11	0.08	1255 + 1113
3	.2250	.1679	.1965	.1893	2.33	1.01	1165 + 1220
4	.1755	.2483	.2119	.1874	4.22	2.47	1352 + 1251
→ 5	.2618	.1373	.1695	.1838	3.81	4.06	1178 + 1223
							12354

Other methods of controlling variation are also available.

Let  $X$  and  $Y$  have means  $\mu_X$  and  $\mu_Y$ , respectively. Suppose that  $\mu_X$  is known but  $\mu_Y$  is to be estimated. One estimate is  $Y$ , another is  $Z = Y + c(X - \mu_X)$  for which  $\text{var}(Z) \leq \text{var}(Y)$  if

$$(4) \quad c \leq -2 \text{ cov}(X, Y) / \text{var}(X) .$$

Consider the airline reservation problem again and let  $X$  denote the sample intercall time,  $\mu_X = 1/\lambda$  and  $c = 1$ . The choice of  $c$  is based on the observation that if  $X - \mu_X$  is positive the intercall times in a replication are above average and, therefore, congestion and waiting time are below average.

Table 3 presents the results of using intercall time as a control variate. The extent of variance reduction is evident.

Table 3.  
Sequential Estimation of Mean Waiting Time  
Using a Control Variate  
 $d = 0.025$ , significance level = 0.05

k	$Z_k$	$\bar{Z}_k$	$s_k^2 (Z)$ ( $10^{-4}$ )	$kd^2/t_{k-1}^2$ ( $10^{-4}$ )	No. of Completions
1	0.2100	0.2100			2651
2	.1648	.1874	10.22	0.08	2497
3	.1760	.1836	5.54	1.01	2781
4	.1528	.1759	6.07	2.47	2500
5	.1624	.1732	4.91	4.06	2629
6	.2079	.1790	5.94	5.64	2550
→ 7	.2134	.1839	6.64	7.31	2595
					18203

When comparing results on experiments with different inputs, variance reduction is again possible. These range from using common seeds for corresponding streams to varying the number of observations collected on each run [3]. For example, suppose that one wants to measure the reduction in mean waiting time that accrues when the number of reservationists increases from 6 to 7. Moreover, the accuracy required is  $d = 1/60$  minutes or 1 second.

Table 4 shows the results when common seeds are used for corresponding streams on corresponding runs. Since

$$(5) \quad s_3^2(X) = 4.4 \times 10^{-4} \qquad s_3^2(Z) = 2.10 \times 10^{-4}$$

Table 4

Sequential Estimation of Mean Waiting Time Difference

 $d = 1/60$  minutes, significance level = 0.05

k	$X_k$ m=6	$Z_k$ m=7	$Y_k =$ $X_k - Z_k$	$\bar{Y}_k$	$s_k^2(Y)$ $10^{-4}$	$kd^2/t_{k-1}^2$ $10^{-4}$
1	0.1884	0.0628	0.1256	0.1256		
2	.1575	.0411	.1164	.1210	0.42	0.03
→ 3	.1976	.0687	.1289	.1236	0.42	.45

variance reduction is estimated to be

$$(6) \quad [s_3^2(X) + s_3^2(Z)]/s_3^2(Y) = 15.5,$$

impressive by most standards.

In some simulation settings it is not possible to match seeds or to induce the necessary correlation between runs to effect a variance reduction. This is especially true when comparing the results of radically different experiments. Here one may have to settle for independent replications, however, variance reduction can still occur. Consider two experiments with outputs  $X$  and  $Z$  and sample sizes per replication of  $n_x$  and  $n_z$ . Let  $\text{var}(X) = \sigma_x^2/n_x$  and  $\text{var}(Z) = \sigma_z^2/n_z$  under the assumption that one is able to create independent observations within each replication [2, 4]. Let  $c_x$  and  $c_z$  denote the unit costs of collecting and processing observations in each replication. If one wants to achieve a specified variance  $V = \sigma_x^2/n_x + \sigma_z^2/n_z$  for  $Y = X - Z$  on each replication then  $n_x$  and  $n_z$  should be selected so



that

$$(7) \quad r = n_x/n_z = r_1/r_2$$

$$r_1^2 = \sigma_x^2/\sigma_z^2 \quad r_2^2 = c_x/c_z$$

Using (7) with  $n_x + n_z = n$  instead of  $n_x = n_z = n/2$  leads to a saving in computing cost of  $(r_1 - r_2)^2 / (1 + r_1^2)(1 + r_2^2) \times 100$  percent.

In preliminary runs of the simulation for  $m = 6$  and  $7$  we estimated  $\hat{\sigma}_x^2/\hat{\sigma}_z^2 = 5.5$  and  $\hat{c}_x/\hat{c}_z = 0.95$  so that  $r = 2.41$ . Ten replications of each experiment were run with  $n_x = 600$  and  $n_z = 250$ . Upon computation of the appropriate terms the estimated saving in computer time needed to achieve the resulting variance for  $\bar{Y}_k = \bar{X}_k - \bar{Z}_k$  was about one third. From this one has to deduct the cost of the two preliminary runs; but that cost was incidental.

The methods of variance reduction discussed here represent a few among many techniques. All exploit the structure of the individual problems to a marginal extent only. However, methods do exist that exploit the properties of individual problems in such a way that substantial variance reductions are possible. These are discussed in [3, Secs 11.2 - 11.3].

5. References

1. Chow, Y. S. and H. Robbins, "On the Asymptotic Theory of Fixed Width Sequential Confidence Intervals for the Mean," Ann. Math. Stat., Vol. 36, 1965, pp. 457-462.
2. Crane, M.A. and D. L. Iglehart, "Simulating Stable Stochastic Systems I: General Multiserver Queues," J.A.CM, Vol, 21, No. 1, January 1974, pp. 103-113.
3. Fishman, G. S., Concepts and Methods in Discrete Event Digital Simulation, Wiley, 1973.
4. Fishman, G. S., "Statistical Analyses of Queueing Simulations," Man. Sci., Vol. 20, No. 3, November 1973, pp. 363-369.
5. Fishman, G. S., "Achieving Specified Accuracy in Simulation Output Analysis, Technical Report No. 74-4, Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 1974.
6. I.B.M., General Purpose Simulation System V User's Manual, 5734-XS2(OS), 5736-X53(DOS), White Plains, New York, 1971.
7. Page, E. S., "On Monte Carlo Methods in Congestion Problems II: Simulation of Queueing Systems," Oper. Res., Vol. 13, No. 2, 1965, pp. 300-305.
8. Potomac Research Group, Fire Support Requirements Methodology Study, Task 1, Phase 1, Verbal and Mathematical Description of the Marine Fire Support Systems, April 1975.
9. Potomac General Research Group, Fire Support Requirements Study, Task 2 Phase I, Review of Theories and Techniques Applicable to Marine Fire Support, June 1975.
10. Starr, N., "The Performance of a Sequential Procedure for the Fixed width Interval Estimation of the Mean," Ann. Math. Stat., Vol. 37, 1966, pp. 36-50.
11. Vaucher, J. G. and P. Duval, "A Comparison of Simulation Event List Algorithms," Comm. ACM, Vol. 18, No. 4, April 1975, pp. 223-230.
12. Wyman, F. P., "Improved Event-Scanning Mechanisms for Discrete Event Simulations," Comm. ACM, Vol. 18, No. 6, June 1975, pp. 350-353.

APPENDIX 8

STATISTICAL ANALYSIS OF SIMULATIONS

# STATISTICAL ANALYSIS OF SIMULATIONS\*

by

Donald L. Iglehart  
Stanford University

## 1. Introduction

The simulation of a stochastic system should be viewed as a statistical experiment. Just as in classical statistics observations are taken with an eye toward making statistical inferences about some unknown parameters associated with the system being simulated. Most simulations are vastly more complicated than the experiments which are analyzed by classical methods of statistics. This leads to two problems: the computer time required to run the simulation long enough to obtain the desired statistical precision can be very expensive and the statistical methodology available for analyzing the results very scanty. These two problems in many cases have prevented simulators from presenting a convincing statistical analysis of the output of their simulations.

In the last three years a statistical methodology has been developed for analyzing the output of the class of regenerative simulations. Our goal in this paper is to present the highlights of the regenerative method. The general problem of analyzing the output of simulations and the basic ideas

---

\*Preparation of this paper was supported by the Office of Naval Research under contract N00014-72-C-0086 (NR-047-106) with Control Analysis Corporation, Palo Alto, California.

behind the regenerative method are discussed in Section 2. In Section 3 the probabilistic structure in the simplest case, Markov chains, is treated. The statistical problem and confidence intervals are dealt with in Section 4. Discrete time methods for reducing computational time are covered in Section 5 and quantile estimation in Section 6. Selecting among competing systems is the subject of Section 7. A numerical example is discussed in Section 8. Approximation techniques to be used when the regenerative structure is absent is the subject of Section 9 and additional technical remarks are made in Section 10. The reference list contains all papers dealing with the regenerative method known to the author, many of which will not be referred to in this paper.

## 2. The Problem and Regenerative Method

Suppose  $\{\underline{X}(t) : t \geq 0\}$  is the vector-valued output of a discrete-event stochastic simulation. Assume that we know that  $\underline{X}(t)$  "approaches a steady-state" as  $t \rightarrow \infty$ ; that is,  $P\{\underline{X}(t) \leq \underline{x}\} \rightarrow P\{\underline{X} \leq \underline{x}\}$  as  $t \rightarrow \infty$ , where  $\underline{X}$  can be viewed as the "steady-state" of the stochastic system. This type of convergence is known as weak convergence and will be written  $\underline{X}(t) \Rightarrow \underline{X}$ . The objective of many simulators is to estimate  $E\{f(\underline{X})\} \equiv r$ , where  $f$  is a specified real-valued function. Examples of  $f$  functions of interest will be given later. Ideally, we would like to produce both point and interval estimates for  $r$ .

The classical method of handling this problem runs something like this. A "good" initial state  $\underline{x}_0$  is selected and the simulation allowed

to run  $T_0$  time units "until the initial transient wears off." Then the simulation is allowed to run for an additional  $t$  units of time. Finally, the point estimate

$$\hat{r}(t, T_0, x_0) = t^{-1} \int_{T_0}^{T_0+t} f[X(s)] ds$$

is given for  $r$ . No confidence interval is given. This method is unsatisfactory for at least three reasons. No guidance is given for selecting  $x_0$  and  $T_0$  and no confidence interval is obtained for  $r$ .

The stochastic processes of concern in this paper are regenerative processes. A regenerative process  $\{X(t) : t \geq 0\}$  with state space  $R^k$ ,  $k$ -dimensional Euclidean space, is loosely speaking a stochastic process which starts from scratch at an increasing sequence of regeneration times  $\{\beta_i : i \geq 1\}$ . That is, between any two consecutive regeneration times  $\beta_i$  and  $\beta_{i+1}$ , say, the portion  $\{X(t) : \beta_i \leq t < \beta_{i+1}\}$  of the regenerative process is an independent, identically distributed replicate of the portion between any other two consecutive regeneration times. However, the portion of the process between time 0 and  $\beta_1$ , while independent of the rest of the process, is allowed to have a different distribution. For complete details on the construction of these processes consult [6].

The regenerative property is an extremely powerful tool for obtaining analytical results for the process  $\{X(t) : t \geq 0\}$ . Before stating these results, we first introduce some notation and make a few assumptions. Let  $\alpha_i$  denote the time between the  $i^{\text{th}}$  and  $(i+1)^{\text{th}}$  regeneration times, that is,  $\alpha_i = \beta_{i+1} - \beta_i$ ,  $i \geq 1$ , and assume  $E[\alpha_i] < \infty$ . Let  $F$  denote

the common distribution function of the  $\alpha_i$ 's. We shall say that  $F$  is arithmetic with span  $\lambda$  if it assigns probability one to a set  $\{0, \lambda, 2\lambda, \dots\}$  for some  $\lambda > 0$ . For our simulation applications we shall assume that the process  $\{\underline{X}(t) : t \geq 0\}$  is piece-wise constant, right-continuous, and makes only a finite number of jumps in each finite time interval. Then if  $F$  is not arithmetic, it is known that  $\underline{X}(t) \Rightarrow \underline{X}$  as  $t \rightarrow \infty$ ; i.e., there exists a random vector  $\underline{X}$  such that the  $\lim_{t \rightarrow \infty} P\{\underline{X}(t) \leq \underline{x}\} = P\{\underline{X} \leq \underline{x}\}$  for every  $\underline{x} \in R^k$  at which  $P\{\underline{X} \leq \underline{x}\}$  is continuous. On the other hand, if  $F$  is arithmetic with span  $\lambda$ , then there exists a random vector  $\underline{X}$  such that  $\underline{X}(n\lambda) \Rightarrow \underline{X}$  as  $n \rightarrow \infty$ .

Now let  $f : R^k \rightarrow R^1$  be a nice (measurable) function and define

$$Y_i = \int_{\beta_i}^{\beta_{i+1}} f[\underline{X}(s)] ds.$$

The goal of our simulation is to estimate  $E\{f(\underline{X})\}$ . A confidence interval for this quantity may be obtained through application of the following two propositions. The first follows from the structure of regenerative processes and the second is proved in [6].

(2.1) PROPOSITION: The sequence  $\{(Y_i, \alpha_i) : i \geq 1\}$  consists of independent and identically distributed random vectors.

(2.2) PROPOSITION: If  $E\{|f(\underline{X})|\} < \infty$ , then

$$E\{f(\underline{X})\} = E\{Y_1\}/E\{\alpha_1\}.$$

This regenerative structure is present for GI/G/s queues in light traffic ([4]) and positive recurrent Markov chains ([5]) and semi-Markov processes. The simplest case of discrete time Markov chains will be discussed in the next section.

We shall show in Section 4 how these two facts may be used to obtain a confidence interval for  $E(f(\underline{X}))$ .

### 3. Probabilistic Structure: Markov Chain Case

Suppose we are interested in simulating a discrete time, irreducible, aperiodic, positive recurrent Markov chain  $\{X_n : n \geq 0\}$  with finite (or countable) state space,  $E$ . Such a Markov chain will possess a stationary (steady-state) distribution:

$$\lim_{n \rightarrow \infty} P(X_n = j) = \pi_j = P(X = j), \quad \text{for all } j \in E$$

where  $\pi_j > 0$  and  $\sum_{j \in E} \pi_j = 1$ . Let  $f : E \rightarrow (-\infty, \infty)$ . We wish to estimate

$E(f(\underline{X})) = \sum_{j \in E} f(j)\pi_j$ . Possible functions  $f$  of interest are the following:

- i) if  $f(i) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$ ,  $j \in E$ , then  $E(f(\underline{X})) = \pi_j$ ;
- ii) if  $f(i) = \begin{cases} 1 & i \geq j \\ 0 & i < j \end{cases}$ ,  $j \in E$ , then  $E(f(\underline{X})) = P(X \geq j)$ ;
- iii) if  $f(i) = i^p$ ,  $p > 0$ , then  $E(f(\underline{X})) = E(X^p)$ ;
- iv) if  $f(i) = c_i = \text{cost of being in state } i$ , then  $E(f(\underline{X})) = \sum_{j \in E} c_i \pi_j$   
= stationary expected cost/period.



A typical sample-path of the Markov chain (M.c.) looks like this

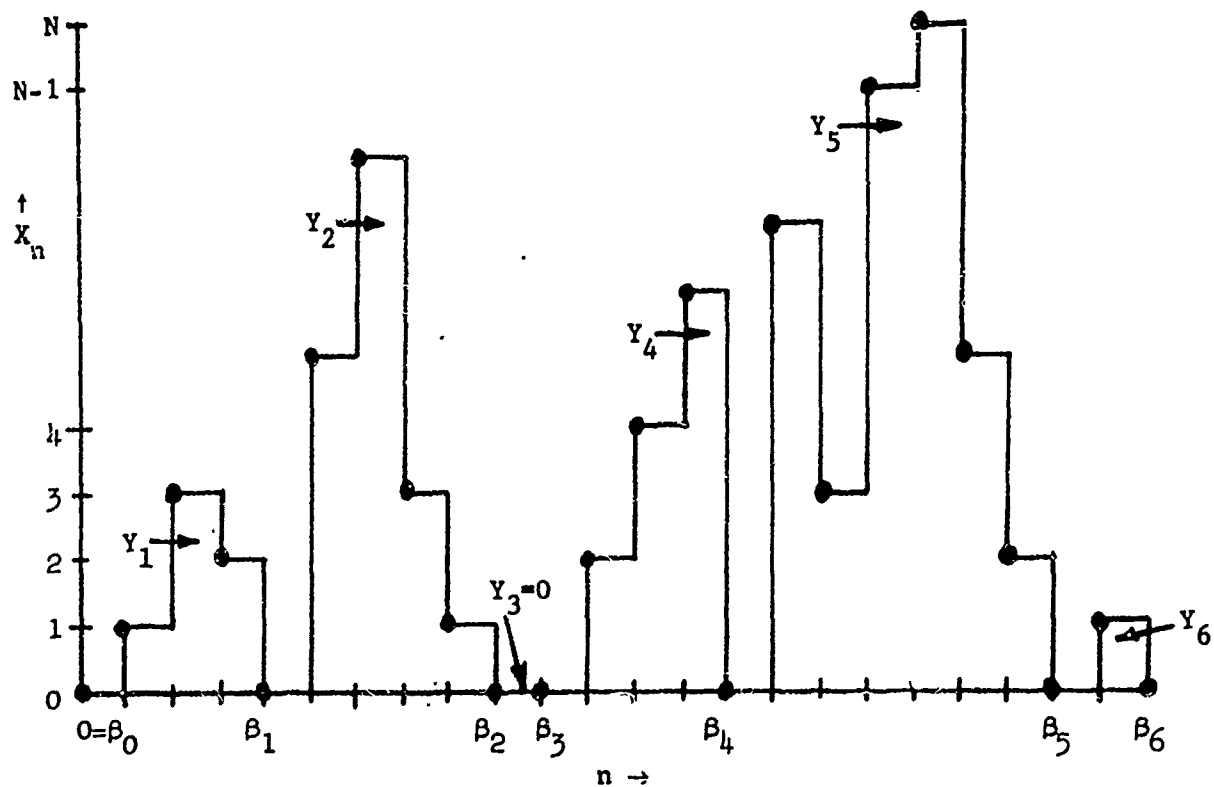


Figure 1

Here  $E = \{0, 1, 2, \dots, N\}$ ,  $f(x) = x$ , and  $\beta_i = \text{time of the } (i+1)^{\text{st}} \text{ visit to state } 0$ . Since the M.c. is positive recurrent, there will be an infinite number of visits to 0 (or any other state for that matter).

#### 4. Statistical Problem and Confidence Intervals

We are given the following observations:  $\{X_k = (Y_k, \alpha_k), 1 \leq k \leq n\}$ , i.i.d., with  $E\{X_1\} = \mu = (\mu_1, \mu_2)$ ,  $\mu_2 \neq 0$ .

PROBLEM: Estimate  $E\{Y_1\}/E\{\alpha_1\} = r = \mu_1/\mu_2$ .

Let  $\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$ ,  $\bar{\alpha} = \frac{1}{n} \sum_{i=1}^n \alpha_i$ , the sample means of the  $Y_i$ 's and  $\alpha_i$ 's. Let  $s_{11} = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{Y})^2$ ,  $s_{22} = \frac{1}{n-1} \sum_{i=1}^n (\alpha_i - \bar{\alpha})^2$ , and  $s_{12} = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{Y})(\alpha_i - \bar{\alpha})$ .

To obtain a confidence interval for  $r$  we need to prove a central limit theorem. Let  $Z_k = Y_k - r\alpha_k$ ,  $k \geq 1$ , and  $\sigma^2 = E\{Z_k^2\}$ . Note that the  $Z_k$ 's are i.i.d. and  $E\{Z_k\} = 0$  from Propositions 1 and 2. Hence if  $0 < \sigma^2 < \infty$ , then  $\sum_{k=1}^n Z_k / \sigma n^{1/2} \Rightarrow N(0,1)$  as  $n \rightarrow \infty$ , where  $N(0,1)$  is a mean zero, variance one normal random variable. By a slight manipulation of this result we obtain

(3.1) PROPOSITION: If  $0 < \sigma^2 < \infty$ , then as  $n \rightarrow \infty$

$$\frac{n^{1/2}[\bar{Y}(n)/\bar{\alpha}(n) - r]}{\sigma/E\{\alpha_1\}} \Rightarrow N(0,1) .$$

This result yields a confidence interval for  $r$  provided we can estimate  $\sigma/E\{\alpha_1\}$ . A variety of estimates have been studied and are reported on in [14]. The most naive estimate of  $\sigma/E\{\alpha_1\}$  is

$$s = [s_{11} - 2(\bar{Y}/\bar{\alpha})s_{12} + (\bar{Y}/\bar{\alpha})^2 s_{22}]^{1/2}/\bar{\alpha} .$$

For large samples (big  $n$ ), this estimate seems to give good results.

## 5. Discrete Time Methods for Continuous Time Processes

Assume now that the regenerative process is either a continuous time Markov chain or semi-Markov process. Then a straightforward application of the regenerative method would require the generation of random holding times (exponential in the case of Markov chains) in the various states. This is expensive in terms of computing time. Here is a simple idea which avoids this time consuming complication; see [13] for details.

Assume the mean holding time in state  $i$  is  $m(i)$ . Replace these random holding times by a constant holding time of length one. Denote the embedded discrete-parameter Markov chain (the so-called jump chain) by  $\{X_n : n \geq 0\}$  and the times of visits of this chain to a fixed state (the return state) by  $\{\beta_n : n \geq 0\}$ . Form for  $k \geq 1$

$$y_k^1 = \sum_{n=\beta_{k-1}}^{\beta_k-1} f(X_n) m(X_n) ,$$

$$y_k^2 = \sum_{n=\beta_{k-1}}^{\beta_k-1} m(X_n) .$$

Then using the same method developed above, we have as  $n \rightarrow \infty$

$$\frac{n^{1/2}[\bar{Y}^1(n)/\bar{Y}^2(n) - r]}{\sigma_1/E\{Y_1^2\}} \Rightarrow N(0,1)$$

where  $\sigma_1^2 = E\{(Y_1^1 - rY_1^2)^2\}$ . The constant  $r$  is the parameter from the original process which we set out to estimate. This central limit theorem provides a confidence interval for  $r$  and only requires the generation of uniform  $[0,1]$  random variables. Furthermore the method is statistically more efficient since it has been shown that  $\sigma_1/E\{Y_1^2\} < \sigma/E\{\alpha_1\}$ . Here  $\alpha_1$  and  $\sigma$  are defined in terms of the original continuous time process.

## 6. Quantile Estimation

Instead of estimating  $E\{f(X)\}$  we might wish to estimate the quantiles of the distribution function of  $f(X)$ . Here we shall assume  $X$  is a scalar random variable and take  $f(x) = x$ . Then our problem is to estimate the quantiles of the distribution function,  $F$ , of  $X$ . The  $p^{\text{th}}$  quantile  $Q(p)$ , of  $F$  is defined to be

$$Q(p) = \inf\{x : F(x) \geq p\}, \quad 0 < p < 1.$$

Of course in nice cases  $Q(p) = F^{-1}(p)$ . As in classical statistics, the problem of estimating quantiles is significantly more difficult than estimating moments.

From a practical point-of-view the problem of estimating quantiles is quite important. Suppose the simulator is designing a computer system and wants to determine the size of a certain memory. He may then want an

estimate of the .95-quantile of the stationary distribution of the number of words stored in the memory. Similarly, in designing the waiting room for a complex queueing system the quantity of interest may be the .90-quantile of the stationary waiting time of a customer. For inventory models we may wish to know the .95-quantile of the inventory level in order to assign storage capacity.

In this section we shall discuss the sample quantile approach to estimating quantiles; for complete details see [15]. Since our desire is to produce a confidence interval for quantiles, we naturally begin by proving a c.l.t. Suppose we agree to run the simulation for  $n$  cycles. Assume for simplicity that  $\beta_1$  has the same distribution as  $\alpha_1$ . (This will not affect any of our limit theorems and is generally the case in practice.) Then we only simulate to time  $\beta_n$ . Let  $1_A$  be the indicator function of the set  $A$ . Then for real  $x$  the function

$$F_n(x) = \beta_n^{-1} \int_0^{\beta_n} 1_{(-\infty, x]}(X(s)) ds$$

can be viewed as the empirical distribution of the regenerative process  $\{X(t) : t \geq 0\}$ . We define the sample quantile,  $Q_n(p)$ , based on  $n$  cycles by the relation

$$Q_n(p) = \inf\{x : F_n(x) \geq p\}, \quad 0 < p < 1.$$

Our next task is to prove a c.l.t. for  $Q_n(p)$ . Select a particular  $p$ ,  $0 < p < 1$ . As in the classical theory, we shall need to make some regularity assumptions on  $F$ , the distribution function of  $X$ , whose quantiles we are attempting to estimate. We shall assume that

$$(6.1) \quad Q(p) = F^{-1}(p) ;$$

$$(6.2) \quad F'(Q(p)) \text{ exists and is positive and finite; and}$$

$$(6.3) \quad F''(x) \text{ exists and } |F''(x)| \leq M < \infty \text{ for } x \text{ in} \\ \text{some neighborhood of } Q(p) .$$

The force of (6.1) - (6.3) is to permit a Taylor series expansion with remainder of  $F(x)$  in the neighborhood of  $Q(p)$ . If the state-space of  $\{X(t) : t \geq 0\}$  is discrete, these assumptions obviously do not hold. The stationary distribution  $F$  will be purely discrete in this case, jumping only at the states of the process. In this case the conditions (6.1) - (6.3) are blatantly violated. Here is a way around this difficulty.

Suppose for sake of discussion that  $E = \{0, 1, \dots, N\}$ . Define a new distribution function  $\tilde{F}$  from  $F$  by linear interpolation between the jumps of  $F$ . For  $-1 \leq x \leq N$ ,

$$\tilde{F}(x) = F([x]) + (x - [x]) [F([x] + 1) - F([x])] .$$

The new distribution function  $\tilde{F}$  is illustrated in Figure 2 for the case  $N = 5$ . The conditions (6.1) - (6.3) are satisfied for the distribution  $\tilde{F}$  except in those rare instances when  $\tilde{Q}(p)$  is one of the integers of  $I$ , where  $\tilde{Q}(p) = \tilde{F}^{-1}(p)$ . In the latter case left and right-hand derivatives of  $\tilde{F}$  exist at the point  $\tilde{Q}(p)$ . We shall define  $\tilde{F}_n$  and  $\tilde{Q}_n(p)$  in a similar way based on  $F_n$  of (3.1). Clearly

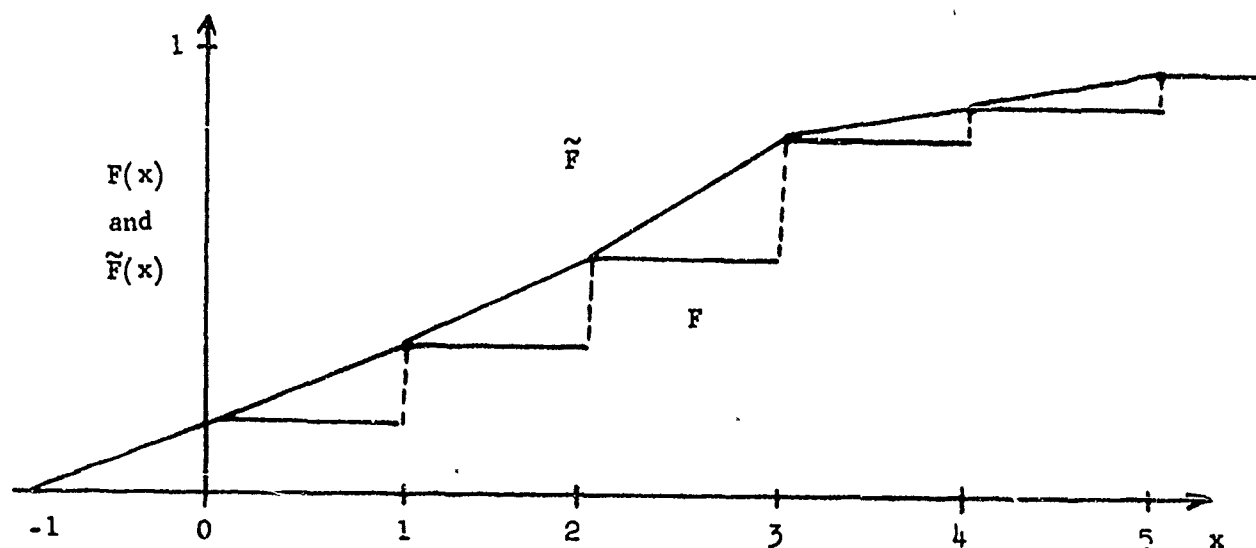


Figure 2

$Q(p) - 1 < \tilde{Q}(p) \leq Q(p)$  and  $Q_n(p) - 1 < \tilde{Q}_n(p) \leq Q_n(p)$ . Assume from here on that the transition from  $F$  to  $\tilde{F}$  has been made when necessary. The basic c.l.t. on which our confidence intervals shall be based is contained in

(6.4) PROPOSITION: If conditions (6.1) - (6.3) hold and  $E\alpha_1^{2+\epsilon} < \infty$  for some  $\epsilon > 0$ , then as  $n \rightarrow \infty$

$$\frac{n^{1/2}[Q_n(p) - Q(p)]}{\sigma(Q(p))/E\{\alpha_1\} F'(Q(p))} \Rightarrow N(0, 1),$$

where  $\sigma^2(x)$  is the variance of a certain random variable.

The problem of estimating the constant  $\sigma(Q(p))/E\{\alpha_1\} F'(Q(r))$  is considerably more complicated than was the corresponding problem in Section 4 and hence will be omitted here. The c.l.t. above plus this estimate yields a confidence interval for  $Q(p)$  in the usual way.

#### 7. Selecting the Best System

Our concern in this section is developing a methodology which can be used in conjunction with the regenerative method to compare the performance of  $k$  ( $\geq 2$ ) systems which are being simulated; for details of the material in this section see [16]. A convenient situation to keep in mind is  $k$  alternative system designs which are being considered. For sake of discussion suppose these  $k$  designs each give rise to a Markov chain  $\{X_n^i : n \geq 0\}$ ,  $i = 1, 2, \dots, k$ , which we will simulate. Suppose our measure of system performance for the  $i^{\text{th}}$  system is  $r_i = E\{f(X^i)\}$ , the expected value of some given function  $f$  of the "steady-state" random variable  $X^i$ . For example, if the system being simulated is a queue, we might wish to base system performance on the expected number of customers waiting. Our goal is to select from the  $k$  systems the one with the largest (or smallest) value of  $r_i$ .

As we do not know the values of the  $r_i$ 's, we shall have to simulate the  $k$  systems and estimate the  $r_i$ 's.

Consider first the special case of  $k = 2$ . In this case the simulator desires to compare  $r_1$  and  $r_2$ . If system 1 is presently in operation and system 2 is a proposed improvement, he may wish to test



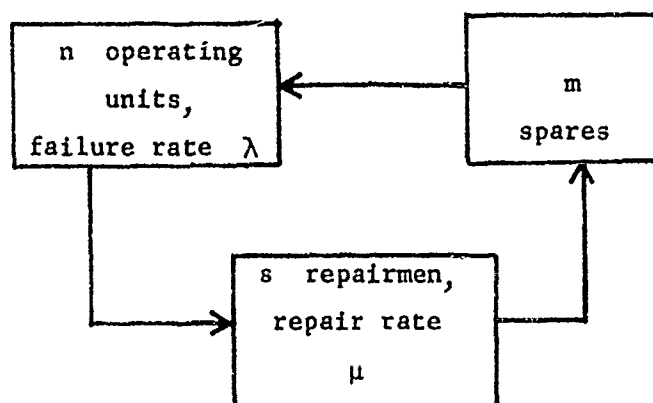
whether  $r_1 = r_2$  or  $r_1 < r_2$  ( $r_1 > r_2$ ,  $r_1 \neq r_2$ ). Tests of hypotheses are indicated in [16] which are of fixed level  $\alpha$ .

For the general case of  $k \geq 2$ , suppose we are interested in selecting the system with the largest  $r_i$ . We begin by specifying two numbers  $P^*$  and  $\delta^*$ . Our goal will be to select with probability  $P^*$  the system with the largest  $r_i$ , whenever that value of  $r_i$  is separated by at least  $\delta^*$  from the other  $r_i$ 's. Because certain variances are unknown and have to be estimated there is no fixed sample size procedure that will guarantee the above goal. Two procedures are given in [16]. The first procedure is sequential and the second two-stage. The sequential procedure has a stopping rule based on estimates of certain variances. The two-stage procedure uses the first stage to estimate these variances. The length of the second stage is then determined by these variance estimates.

Tests of hypotheses ( $k = 2$ ) and the two selection procedures ( $k = 3$ ) were carried out for the classical repairman model. The numerical results for the tests of hypotheses were in good agreement with the theory. Both the sequential and two-stage methods attained the required goals for the probability of correct selection,  $P^*$ . The estimates used in the two methods must be chosen properly in order to both attain the desired statistical results and minimize the computational time required to carry out the simulations.

## 8. Numerical Example

Consider the classical repairman problem:



Let  $X(t) = \#$  units waiting for or undergoing repair at time  $t$ . Assume failure and repair times are exponentially distributed. Then  $\{X(t): t \geq 0\}$  is a birth-death process with state space  $E = \{0, 1, \dots, m+n\}$ , birth parameters  $\lambda_i = (n - [i-m]^+) \lambda$ , and death parameters  $\mu_i = (i \wedge s) \mu$ . Also  $P\{X(t) = j\} \rightarrow P\{X = j\} = \pi_j$ , for all  $j \in E$ . Some simulation results for  $E\{f(X)\}$  are shown in TABLE 1.

The quantiles of  $X$  were also estimated. Here are some typical results for the same parameter values used in Table 1. Because of the discrete state space the transition from  $F$  to  $\tilde{F}$  has been made. The estimates in Table 2 are the sample means of 100 replications each of length 200 cycles. These estimates yielded confidence intervals consistent with the theory.

The discrete time method proposed in Section 5 was used for this repairman problem when  $f(x) = x$ . For the continuous time simulation the theoretical value of the constant  $\sigma/E\{\alpha_1\}$  in Proposition 3.1 is

TABLE 1

## Simulation Results

( $\lambda = 1$ ,  $\mu = 4$ ,  $s = 3$ ,  $n = 10$ ,  $m = 4$ ; run length 300 cycles)

Level of Confidence = 90%

Parameter	Theoretical Value	Point Estimate	Confidence Interval
$E\{X\}$ = expected # down	3.471	3.406	[3.205, 3.607]
$E\{X^2\}$	17.278	16.844	[15.094, 18.594]
$P\{X > m\}$ = prob. less than $n$ operating	.306	.294	[.260, .328]
$P\{X > s\}$ = prob. of a queue at repair facility	.438	.429	[.393, .465]
$E\{[s-X]^+\}$ = expected # of idle repairmen	.678	.705	[.637, .773]
$P\{X > 0\}$ = prob. at least one unit down	.939	.930	[.919, .942]
$P\{X = 0\}$ = prob. no unit is down	.061	.070	[.058, .081]

TABLE 2  
Quantile Estimation

p	true value of $Q(p)$	true value of $\sigma/E(\alpha_1)F'$	estimate of $Q(p)$	estimate of $\sigma/E(\alpha_1)F'$
.50	2.608	2.151	2.575	2.125
.75	4.507	2.887	4.439	2.897
.90	6.238	3.686	6.165	3.451

1.9553, whereas the theoretical value of  $\sigma_1/E\{Y_1^2\}$  for the discrete method is 1.9022. Thus only a 2.6% statistical saving is realized. However, the continuous time simulation required 1.79 minutes to replicate 100 cycles 100 times and the discrete time simulation only 0.99 minutes, a computational saving of about 44.7%. In all examples run thus far the statistical saving is minor, but the computational saving significant.

Because of the additional complications of the selection procedures discussed in Section 7 numerical results will not be described here. Numerical examples can be found in [16].

#### 9. Approximation Techniques

A discrete-event simulation may not possess the nice regenerative property exploited above. Furthermore, even if it does possess the regenerative property the return states may be hard to identify or may not occur frequently enough to use the regenerative method. Nevertheless, it may still be the case that  $\underline{X}(t) \Rightarrow \underline{X}$  and estimating  $E\{f(\underline{X})\}$  may

be of interest. Here are two approximation techniques which may be employed in this situation; for more details see [7].

In the first method select a fixed state  $\underline{x}$  and construct a small region  $A$  surrounding  $\underline{x}$ . Define  $\beta_1, \alpha_1, Y_1$  in terms of returns to  $A$  rather than  $\underline{x}$ . Compute confidence intervals by old method. Here the pairs  $(Y_1, \alpha_1)$  will not be independent nor identically distributed. But if  $A$  is "small" the distributions should be close and dependence fall off rapidly. This method has been used in [22] with some success for a simulation of a job-shop.

The second method forms a modified process by setting the original process equal to  $\underline{x}$  each time it enters  $A$ . This modified process is then a regenerative process to which the regenerative method can be applied. However, in this case the confidence intervals produced are with respect to the modified process, not the original one.

#### 10. Additional Technical Remarks

In this final section we collect together a number of additional remarks.

1. If confidence interval is desired after a fixed simulation time  $T$  rather than a fixed number of regeneration cycles  $n$ , the method can still be used. Simply use the same formulae with  $n$  replaced by  $N(T)$ , the # of completed cycles by time  $T$ .
2. If two or more regeneration sequences exist, the simulator may choose whichever is more convenient as the lengths of the two confidence intervals are asymptotically of the same length.

3. Suppose that in addition to wanting to estimate  $E\{f(X)\}$  the simulator also wants to estimate  $E\{f(X(10))\}$ , say. Then independent samples of  $f(X(10))$  can be read off after 10 time units elapse after the start of a cycle, the samples being based on non-overlapping portions of the process.

4. The same approach can be used to estimate the asymptotic cost per unit time. Let  $\{C(t) : t \geq 0\}$  be such a cost function and assume that

$$Y_i = C(\beta_{i+1}) - C(\beta_i)$$

and

$$\alpha_i = \beta_{i+1} - \beta_i$$

are i.i.d. Then

$$\frac{C(t)}{t} \rightarrow \frac{E\{Y_1\}}{E\{\alpha_1\}}$$

with probability one.

5. After a run of  $n$  cycles one can obtain a  $100(1-\gamma)\%$  confidence interval of length

$$\approx \frac{c \cdot \phi^{-1}(1 - \gamma/2)}{\sqrt{n}} .$$

The proportionality constant  $c$  can be estimated with a short run, following which the simulator could make tradeoffs between

- i) run length,
- ii) level of confidence, and
- iii) length of confidence interval.

## REFERENCES

- [1] COCHI, B.J. (1973). A stochastic model of interleaved memory systems. Ph.D. Thesis, Department of Electrical Engineering, Stanford University.
- [2] CRANE, M.A. and IGLEHART, D.L. (1973). A new approach to simulating stable stochastic systems. Proceedings of the 1973 Winter Simulation Conference, San Francisco, 264-272.
- [3] CRANE, M.A. and IGLEHART, D.L. (1974). Statistical analysis of discrete-event simulations. Proceedings of the 1974 Winter Simulation Conference, Washington, D.C., 513-521.
- [4] CRANE, M.A. and IGLEHART, D.L. (1974). Simulating stable stochastic systems, I: General multi-server queues. J. Assoc. Comput. Mach. 21 103-113.
- [5] CRANE, M.A. and IGLEHART, D.L. (1974). Simulating stable stochastic systems, II: Markov chains. J. Assoc. Comput. Mach. 21 114-123.
- [6] CRANE, M.A. and IGLEHART, D.L. (1975). Simulating stable stochastic systems, III: Regenerative processes and discrete-event simulations. Operations Res. 23 33-45.
- [7] CRANE, M.A. and IGLEHART, D.L. (1975). Simulating stable stochastic systems, IV: Approximation techniques. Management Sci. 21 1215-1224.
- [8] FISHMAN, G.S. (1973). Statistical analysis for queueing simulations. Management Sci. 20 363-369.
- [9] FISHMAN, G.S. (1974). Estimation in multiserver queueing simulations. Operations Res. 20 72-78.



- [10] FISHMAN, G.S. (1974). Simulation data analysis using continuous time Markov processes. Technical Report No. 74-1, Operations Research and Systems Analysis, University of North Carolina at Chapel Hill.
- [11] FISHMAN, G.S. (1974). Achieving specified accuracy in simulation output analysis. Technical Report No. 74-4, Operations Research and Systems Analysis, University of North Carolina at Chapel Hill.
- [12] GREGORY, M.T. (1975). Discrete event simulation of stable stochastic systems. M.S. thesis, Department of Mathematical Sciences, Clemson University.
- [13] HORDIJK, A., IGLEHART, D.L., and SCHASSBERGER, R. (1975). Discrete time methods of simulating continuous time Markov chains. Submitted for publication.
- [14] IGLEHART, D.L. (1975). Simulating stable stochastic systems, V: Comparison of ratio estimators. To appear in Naval Res. Logist. Quart.
- [15] IGLEHART, D.L. (1975). Simulating stable stochastic systems, VI: Quantile estimation. Submitted for publication.
- [16] IGLEHART, D.L. (1975). Simulating stable stochastic systems, VII: Selecting best system. Submitted for publication.
- [17] LAVENBERG, S.S. and SLUTZ, D.R. (1975). Regenerative simulation of a model of an automated tape library. IBM Research Report RI-1563, San Jose.
- [18] LAVENBERG, S.S. and SLUTZ, D.R. (1975). Regenerative simulation: A tutorial. IBM Research Report RI-1562, San Jose.

- [19] LAVENBERG, S.S. and SHEDLER, G.S. (1975). A queueing model of the DL/I component of IMS. IBM Research Laboratory, San Jose.
- [20] LAW, A.M. (1974). Efficient estimators for simulated queueing systems. ORC 74-7, Operations Research Center, University of California, Berkeley.
- [21] NEVIUS, T.W. (1974). On simulating stable stochastic systems. M.S. thesis, Department of Mathematical Sciences, Clemson University.
- [22] RIVER-GARZA, M. (1974). Labor and machine constrained job shops: A simulation investigation of dispatching rules. Engineer thesis, Department of Industrial Engineering, Stanford University.
- [23] SAUER, C.H. and CHANDY, K.M. (1975). Approximate analysis of central server models. To appear IBM Journal of Research and Development.
- [24] SAUER, C.H. (1975). Simulation analysis of generalized queueing networks. IBM Research Center, Yorktown Heights, N.Y. (to appear in Proc. 1975 Summer Computer Simulation Conference).
- [25] SMITH, A.J. (1974). Performance analysis of computer systems components. Ph.D. thesis, Department of Computer Science, Stanford University.
- [26] TURNER, C.A. (1975). A comparison of ratio estimators for simulating stable stochastic systems. M.S. thesis, Department of Mathematical Sciences, Clemson University.
- [27] WILHEIM, N.C. (1973). Exact models for two spindle disk drives. Ph.D. thesis, Department of Electrical Engineering, Stanford University.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TECHNICAL REPORT NO. 86-16	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Statistical Analysis of Simulations		5. TYPE OF REPORT & PERIOD COVERED Technical
7. AUTHOR(s) Donald L. Iglehart		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Control Analysis Corporation 800 Welch Road Palo Alto, CA 94304		8. CONTRACT OR GRANT NUMBER(s) N00014-72-C-0086
11. CONTROLLING OFFICE NAME AND ADDRESS Operations Research Program Office of Naval Research Arlington, Virginia 22217		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR-047-106
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE July 1975
		13. NUMBER OF PAGES 23
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		16a. DECLASSIFICATION/DOWNGRADING SCHEDULE
18. DISTRIBUTION STATEMENT (of this Report) Approved for public release: distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
19. SUPPLEMENTARY NOTES Prepared for the Fire Support Requirements Methodology Workshop, Naval Postgraduate School, Monterey; August 4-7, 1975.		
20. KEY WORDS (Continue on reverse side if necessary and identify by block number) REGENERATIVE METHOD SIMULATION STATISTICAL ANALYSIS OF SIMULATIONS		
21. ABSTRACT (Continue on reverse side if necessary and identify by block number) This paper contains an expository account of the regenerative method for simulating stable stochastic systems.		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE  
S/N 0103-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

APPENDIX 9  
TARGET GENERATION AND DESIGNATION

## TARGET GENERATION AND DESIGNATION

by Robert G. Hinkle

For purposes of this workshop, Target Generation and Designation have been considered as a unified process of "sensing the presence of enemy units and preparing a ranked dynamic target array". The target array consists of a set of entries representing demands for supporting fire and hence include entries for suppression, preparation fires, interdiction, and fire requirements against suspected enemy positions. Thus, the target array represents the demands placed on the Fire Support System.

The objective of this working group was to consider the questions associated with the analytic generation of these demands and the form they should take for purposes of analytical evaluation of candidate Fire Support Systems or alterations to existing Fire Support Systems.

The approach taken in the working group was to discuss the target generation process in general terms as it relates to the operation, review the current methods for modeling the process, identify areas of weakness in the methodology, discuss some proposed ways of strengthening the methodology, and identify areas for needed research.

Figure 1 is a conceptual flow diagram of the target generation/designation process. The idealized target set is a continually evolving set of targets or firing opportunities which exist under perfect intelligence and evaluation. The target generation system produces a target list which includes some "real targets", some targets which are not on

the idealized list such as false targets or suppression fires which are doomed to be ineffective because the enemy forces are not in the area. There is a dynamic interaction between the idealized target set, the target list and the implementation of the corresponding fire plan. These interactions are caused by, among other things, the time lag between the occurrence of the idealized target set (whose composition changes continually) and the final prepared target list which is an estimate of the idealized target set at some previous point in time. In addition there is a strong interaction of the fire plan implementation and both the ideal and actual target list in that the enemy loses units, pieces of equipment and changes his tactics because of the fire directed against his forces.

Figure 2 is a conceptual model of the analytic counterpart to the "operational" target generation process. A general conclusion of the group was that the exact form of the model and the level of detail represented in the model must be tailored to types of studies for which it is to be used and the type of information being sought from the study. This of course depends upon the decisions being supported by the study.

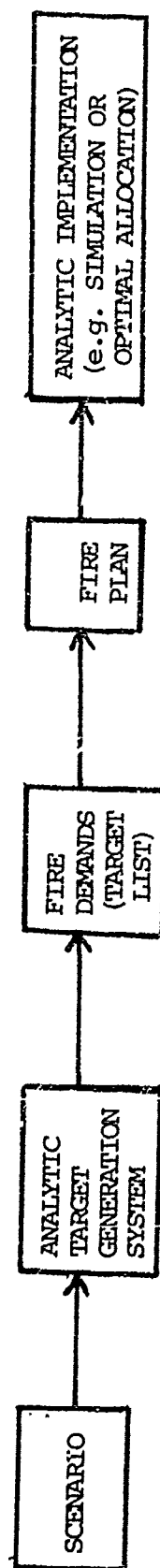


FIGURE 2. FIRE SUPPORT SYSTEM EVALUATION MODEL

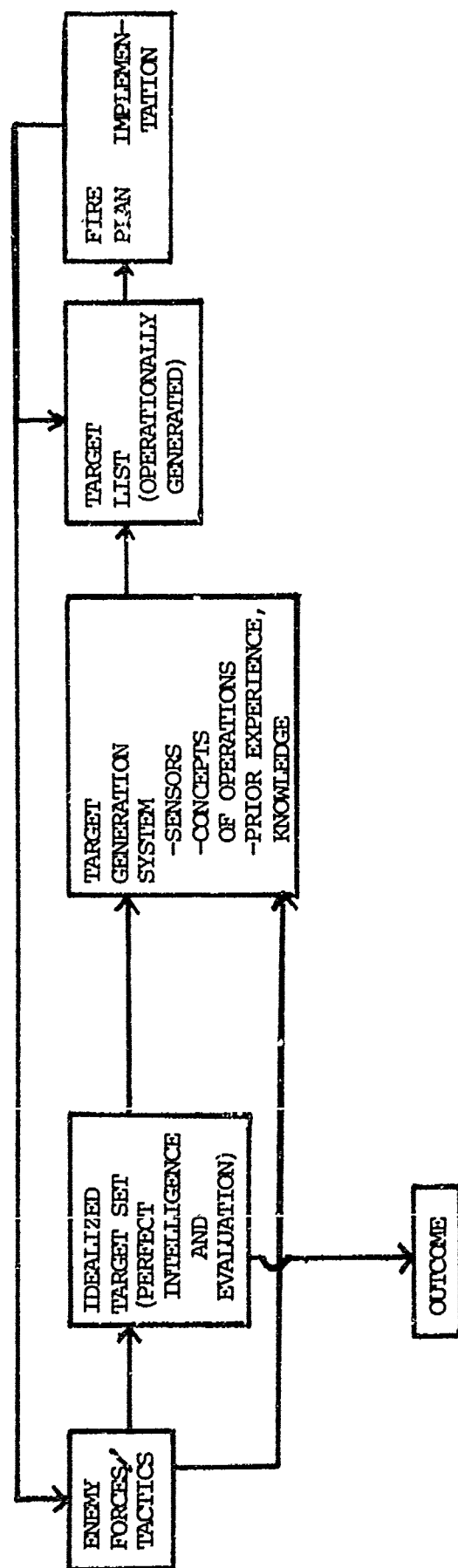


FIGURE 1. DYNAMIC TARGET GENERATION PROCESS



#### ISSUES CONSIDERED

The working group agreed to focus primarily on two issues related to target generation and designation: (1) the adequacy of analytic methods (models) for generating target lists (or fire support requirements) and (2) the need for and possible means of introducing dynamic interaction between the fire support means employed and the target list.

#### TARGET GENERATION MODELS

The target generation model is either a simulation of the operational process which generates target lists or an empirical model which generates target types and quantities based on historical data.

The target generation process consists of the knowledge of the force mission and tactics; knowledge and preception of enemy forces, (order of battle), force disposition, and tactics; and the tactical intelligence system/decision process.

It was the consensus of the working group that the sensor systems are well understood and well modeled. The major area requiring attention is the sensor data processing system which is essentially a human process. Thus, sensor contacts do not themselves constitute a target list. Another area requiring attention is the generation of fire support requirements for harrassment and interdiction, preparation fires, and suppression. Any operational target generation process will generate false targets due to aging of sensor and other intelligence information and due to incorrect assessments of intelligence information.

If one is considering the use of detailed simulation models for fire support systems evaluation, the difficulties of modeling these processes must be resolved otherwise the "noise" introduced by guesses will mask the effects sought in going to the detailed simulation.

#### DYNAMIC INTERACTIONS

The operationally generated target list is a continuously changing list due to the continuous inflow of intelligence information and the assessment of damage done by partial implementation of the current fire plan. In manual war games this dynamic nature of the target list is accounted for by the two-sided interactive nature of the tool. The target list generated is strictly valid only for that specific play of the game and the introduction of different weapons, sensors, or forces (players) will generate a new target list likely to be quite different from others.

In an analytic model of fire support operations it is important that this interaction be represented. The use of a Markov process or Stochastic network model may be a tool which would allow implementation of this process. The idea is that the fire support allocation process be considered at discrete points in time. At a given time there is an ideal target set, an operationally generated target set, and an inventory of remaining fire support assets. The decision maker must make a decision regarding the allocation of assets to targets on the list. As a result of his decision (fire plan) and its implementation, the state of the system will change with certain probabilities to one of the set of possible new states. The process continues for the duration of the operations.

For a given fire support mix the final values of any number of the chosen state variables can be used as the measures of effectiveness.

This concept is by no means well thought-out at this time and hence neither its computational efficiency nor its value is clear.

#### CONCLUSIONS AND RECOMMENDATIONS

The following general conclusions and recommendations are made by the working group.

Sensor models are adequate

Man part of the sensing process is not well understood therefore not well modeled

Ideal target set and target list should be kept distinct

Dynamic interaction between target set (hence the target list) and the Fire Plan Implementation should be modeled

The degree of resolution/aggregation required in the methodology and hence in the target array description is driven by the study question

There does not exist a model (other than the landing force war game) which will produce detailed target lists

New sensors and data processing systems will place increasing demands on Fire Support Allocation Engagement Process

There is a need for threat studies to provide a basis for fire support requirements generation

There is a need for a simulation model of fire support imbedded in model of combined arms

There should be a scientific review of alternating Markov renewal process utilized in Vector-2

An in-depth study of the target generation process should be made, including command, control and communication, intelligence data processing, and inference.

APPENDIX 10  
CHAIRMAN'S REPORT  
TARGET ENGAGEMENT MINIWORKSHOP

CHAIRMAN'S REPORT  
TARGET ENGAGEMENT MINIWORKSHOP  
by A. H. Goettig

On the third day of the Fire Support Methodology Workshop, the participants of the workshop were divided into three miniworkshops to consider specific areas of fire support methodology. The areas covered by the three miniworkshops were:

1. Target Generation and Designation
2. Target Engagement
3. Fire Support System Effectiveness

This report describes the activities of the Target Engagement group. The prime objective of the miniworkshops was to examine the applicability of the techniques described during the presentations of the previous two days to specific methodology problems.

Membership on the group appeared to represent a cross section of the general attendance of the overall workshop, i.e., Marines, Army, University, Contractor and in-house Navy civilians. They were:

- A. Goettig, Chairman (NWC, China Lake)
- J. Bobick (SRI)
- K. Chan (IDA)
- G. Fishman (Univ. of N.C.)
- K. Harrison, LCOL (MCDEC)
- F. Hartman, CAPT (Army CAA)
- D. Osteyee (ONR, Pasadena)
- L. Peters (SRI)
- J. Taylor (NPGS)
- D. Zimmerman (PGRC)

### CHAIRMAN'S COMMENTS

The suppression effects obtained by fire support was the miniworkshop members primary area of interest. The broad findings about suppression effects were:

1. They should be included in fire support methodology.
2. This should be done parametrically if data is lacking.
3. Efforts should continue to obtain data on troop behavior while under fire.

Recommendations in the PGRC report for use of specific techniques were not:

1. Supported by thorough comparisons with existing and evolving techniques.
2. Consistent with the anticipated use of the methodology.
3. Consistent with the degree of uncertainty in the data that would be used.

The PGRC plan emphasized more realism and detail in the methodology. The acknowledged lack of data would normally indicate a need for more aggregation to enable very fast running of programs and exploration of the effects of uncertainty in data.

The process of developing a fire support methodology needs strong and continuing inputs from three kinds of people in addition to the analysts who technically develop the methods. These are:

1. Combat experienced personnel who know what kind of data can be obtained.
2. Study managers who have had experience with utilizing methodology to compute answers to someone's questions.
3. Project managers who will have to make decisions on alternative fire support concepts.

A critical factor in developing a useful methodology is to provide for a continuing input from these three elements to help steer the direction of methodology development.

The target engagement miniworkshop addressed a number of questions with the intent of developing statements relevant to each question which would meet two criteria:

1. Represent the concensus of the group.
2. Provide useful guidance for fire support methodology development.

The questions addressed by the group were:

1. What is the Marine Fire Support Methodology problem?
2. Why is suppression important to include in the methodology?
3. Why isn't suppression adequately included in existing models?
4. What should be done about suppression in the methodology until troop behavior data is available?
5. What sources of troop behavior data appear to be most promising?
6. What does the group think about the methodology development plan contained in the PGRC Report?

In addition to the above six questions the group developed statements about:

1. The fire assessment equation in the Phase I task's report.
2. The need for including environmental parameters in fire support analysis.
3. Efforts to provide greater detail and realism in models and simulations.

This report is structured according to the question or topic addressed by the group. For each question or topic the statement developed by the group is presented and a brief description of the discussion that took place during the development of the statement is also included.



What is the Marine Fire Support Methodology Problem?

Statement: The Marines need a low cost methodology that will enable the selection of a mix of fire support weapons. Mix, in this sense, means both which systems to buy and what quantities of expendable ammunition are needed.

Discussion: The above statement is the chairman's interpretation of the group discussion. In essence it is the same as what was stated in the PGRC report but with emphasis on low cost and explicit recognition that there are two facets to be considered in determining a mix.

Why is suppression important to include in the methodology?

Statement: Suppression and neutralization are the primary things accomplished by fire support, however current methods deal only with the killing or destruction of targets. In determining a mix of weapons comparisons are needed between the suppressive effects of different weapons.

Discussion: The need for including suppression in fire support methodology was generally accepted by the group and answering this question seemed almost academic.

Why isn't suppression adequately included in existing model?

Statement: There is inadequate data on troop behavior when under fire.

Discussion: The group agreed that there is currently no major problem in modeling suppression effects but there is no supportable data on troop behavior when under fire to use in the models. The group realized that this is not a new finding and that efforts are underway that are attempting to develop the data experimentally.

What should be done about suppression in the methodology until troop behavior data becomes available?

Statement: Suppression effects should be treated as a parameter until data becomes available.

Discussion: Perhaps the real question here was whether it is more dangerous to use unsupported estimates or to ignore the problem altogether. In any case we agreed that including suppression effects in a parameterized form is necessary. This would accomplish several things:

1. Require selection of MOEs that would include suppressive effects. If done carefully this should help focus efforts to collect troop behavior data and to design suppression models.
2. Help identify those cases where the uncertainty about suppressive effects has a significant effect on a decision.
3. Provide a quantitative indication of the need for accurate suppression modelling.

What sources of troop behavior data appear to be most promising?

Statement: Combat experience is felt to be the most likely source of convincing data on suppressive effects.

Discussion: Actually replacing the words "most likely source" in the above statement with the words "only source" might better reflect the group's feelings. We were not able to see how convincing data on behavior when exposed to a lethal environment can be obtained without using a lethal environment. Nevertheless, we were not willing to claim that it couldn't be done nor did we want to discourage ongoing efforts that are attempting to get a handle on the problem. In any case when dealing with conceptual systems or systems that have not been used in combat, estimates of the suppressive effects on operators will have to be used.

What does the group think about the methodology development plan contained in the PGRC Phase I Report?

Statements related to this question were developed for each of the recommendations made during M. Zimmerman's briefing on Monday. The recommendations were divided into near-term and far-term. Elaboration on the recommendations are

included in the Phase I Summary report which most of the group had at least scanned, Mr. Zimmerman was a member of our group and also provided explanations of the briefly stated recommendations on the view graphs he had used.

#### NEAR TERM PLAN

##### Target List for Kinematic Analysis

(The group decided this subject was included in the charter of another mini-workshop. A lively interchange about whether it was better to have target lists generated inside a simulation or outside got started but we did not formulate a statement).

##### Combine and Modify the DAFS/CAS and MAF Models

Statement: Before selecting near-term model(s) the following should be done:

- a. Define the Marines near-term questions.
- b. Identify the systems involved and decision criteria.
- c. Identify models that deal with the systems and decision criteria identified in b.
- d. Select model(s).

Discussion: It turns out that this was the procedure intended in the Phase I report but it was not apparent to the group and therefore, the rationale for the recommendation was not apparent. In addition, additional models had been discussed in the workshop briefings that might be candidates and had not been evaluated. The net result is that a more explicit problem definition and decision rationale than is contained in the Phase I report is needed.

##### Determine State-of-the-art of what we know about suppression.

Statement: None

Discussion: The group felt that to a large extent this had been accomplished by people in Army and Navy Programs. Two documents that address the area of suppression and also have rather extensive bibliographies of reports related

to suppression are:

- a. NWC TN 12-72-1, "Background and Approach for Study of Combat Suppression"  
by E. G. Swann dtd May 1972, UNCLASSIFIED
- b. NPGS Masters Thesis in OR, "The Effect of Suppression on the Casualty  
Exchange Ratio," by Edgar C. Johnson dtd Mar 73 (AD 911 883)

People who have been actively working in the area of suppression and modeling  
the development of suppression effects data are:

E. G. Swann	NWC, China Lake	AV 245-3811
Dr. J. Taylor	NPGS	AV 479-2683
Dr. Eugene Dutoit	U.S. Army Infantry School Fort Benning, Georgia	AV 835-2015
Dr. H. Fallin	USAAMSAA, Aberdeen, MD	AV 870-3785
Dr. Seth Bonder	Victor Research Inc., P.O. Box 1505 Ann Arbor, Mich. 28106	(313) 972-9210
Dr. Edgar Johnson	U.S. Army Research Institute 1300 Wilson Blvd. Arlington, VA 22209	AV 224-1694 (202) 694-1694

Additionally a central contact point for the developing Army study program in  
suppression is:

MAJ Edward J. Burke	Combined Arms Combat Development Activity ATTN: ATCA-CCM-1 Fort Leavenworth, Kansas 66027	AV 552-5595
---------------------	---	-------------

## FAR TERM

### Design-Value-Driven Simulation

#### Develop and Apply Algorithmic Programs

Statement: The value of algorithmic and value-driven models has not been clearly demonstrated and commitment to these techniques would be premature. Exploration and comparison of these techniques does appear to be in order.

Discussion: The group recognized that it lacked the in-depth understanding of these techniques that should be represented in either a strong positive or negative endorsement of these techniques. On the other hand it appeared that we were not alone in our lack of understanding and serious scrutiny of these techniques is in order before a decision is made.

#### Scenarios and Concepts

Statement: We endorse the idea of standardizing scenarios (including threats) and concepts of operations as a means of enhancing comparisons between studies and reducing study costs. This should be done regardless of decisions about whether or not to conduct an overall fire support study. We realize updating will be required from time-to-time and that new systems will not necessarily be compatible with established concepts of operations.

Discussion: The group also felt that this recommendation might also provide some help in the area of target lists for specific often-occurring combat situations. Standardizing should not be interpreted as preventing reasonable excursions or deviations if they are done in an explicit manner and are based on data (particularly in the case of threats) rather than conjecture.

### Structure of Combat

Statement: The relationships between fire support and rates of advance are not well supported by data and yet these relationships are key factors in simulations, games and models. Suggested ways for improving confidence in the data were:

1. More detailed simulation.
2. Study of combat experience.
3. Use of data from exercises and maneuvers.
4. Experimentation.

Discussion: We did not reach agreement on what, if anything, could be gained from these various approaches and in addition, there was a variety of opinion on whether absolute values were really critical or whether relative values would suffice in most cases. This was the last topic of the day to be covered by the group and we did not see any way to resolve our positions in a short period of time. It was decided that it was better to present the unevaluated suggestions above rather than not comment at all.

### OTHER TOPICS

#### Fire Assessment Equation

Statement: Refinement in the fire assessment equation in Section 4.7 of Phase I Task I report is needed.

Discussion: One member of the group had examined the Phase I reports in enough detail to determine that more representative treatment of an offset MPI is typically used to compute  $P_k$ 's for artillery fire. This was discussed with an author of the report.

#### Need for Environmental Parameters

Statement: The effects of dust, smoke, haze, etc., will influence the target engagement problem with systems like CLGP and LGB's. These effects should be included in fire support analyses and be supported by data.

Discussion: The group felt limited visibility conditions can have a significant influence on the performance of some weapons as well as the detection and obscuration of targets. Limited visibility conditions occur quite often, either naturally or as a result of combat, and therefore are important to include.

Efforts to provide greater detail and Realism

Statement: Efforts to provide greater detail and realism should be carefully examined with regard to contribution to overall results.

Discussion: The group observed that the proposed plan in the Phase I report placed a lot of emphasis on additional detail, realism and resolution. While there was not unanimous agreement on this statement, the consensus was that there is a tendency to strive for extreme detail in one part of a model while at the same time gross estimates are being used in another part. After developing this statement we realized we had used a lot of words to restate the concise phrase used by Dr. Shubik in his presentation the previous day, i.e., "Relevance vs. Realism."

APPENDIX 11  
CHAIRMAN'S REPORT  
FIRE SUPPORT SYSTEM MIX EFFECTIVENESS  
ANALYSIS WORKSHOP

by Edward W. Girard



Report of Mini-Workshop on Fire Support System  
Mix Effectiveness Analysis

The attendees are listed at the end of this report. The morning session was attended heavily, the afternoon session, sparsely. To keep the deliberations consistent with the main workshop, and free of avoidable and perhaps inconsistent overlap with the other two paralleling sessions, two assumptions were made.

Our concerns are with the non-strategic, non-decisive uses of fire effects.

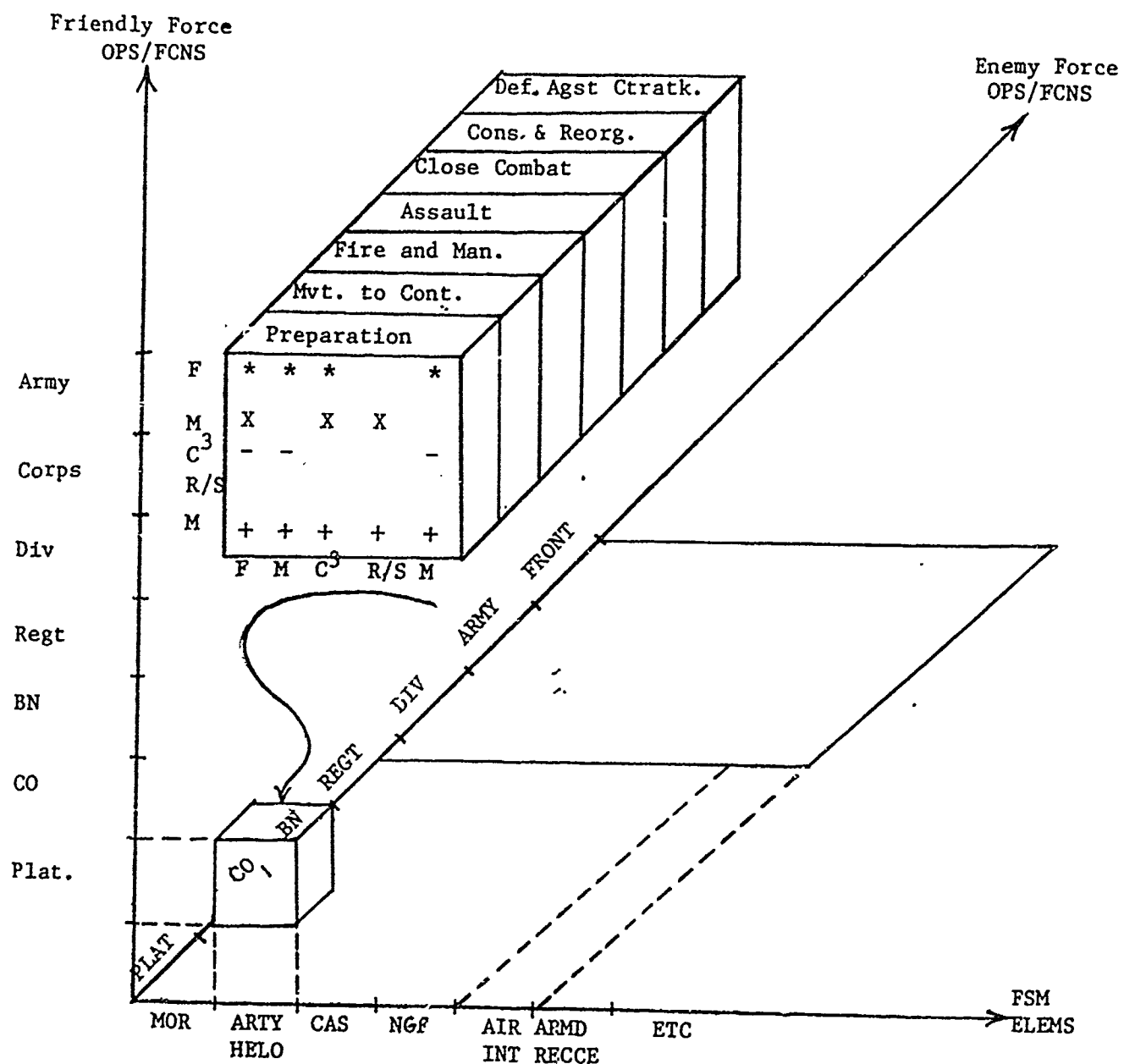
All issues of target definition and engagement analyses are being satisfactorily taken up by their respective mini-workshops.

By way of a prolonged opening statement the chairman sketched two analyses of fire support to show that there remain significant advances to be made in thinking, managing, developing and operating in the realm of fire support. The criteria used were principally two, effectiveness of fire support in lowering battle losses to supported engaged infantry<sup>1</sup>, and attractiveness of fire support as an alternative to increased ground operations in a joint land/air theater force campaign.<sup>2</sup> Both examples served to introduce the main methodological point that fire support mix analysis is a very complicated, painstaking activity because of the multidimensional character of the mix issue in any real problem. This is seen in the figure, where the usually understood component of fire support mix elements is indicated by the horizontal axis in the viewing plane. When a real mix is to be evaluated in support of a real force's operations, support is provided to more than one friendly echelon. MOE's must then be defined with respect to the characteristic missions, scope, tempo, and key operational functions of all of the supported echelons involved. When carrying through a determination of benefit at a given supported echelon, it is also necessary to consider the interplay of its functions with those of the enemy force echelons that are found in opposition. In other words, there is a rather straightforward if tedious logic of producing the benefit of fire support to our friends via imposing a degrading effect of fire on our (their!) foes. Again, when we try to be specific in this matter, we deal with degradation of the enemy echelons' operational functions, with consequent enhancement of those of our supported friends. The beginning of the structure of an effectiveness

---

<sup>1</sup> Edward W. Girard, Structural Approaches to Fire Support Systems Mix Analysis (U), Proceedings of 33rd MORS, USMA, June 1974, Unclassified.

<sup>2</sup> Edward W. Girard, Contributions of Tactical Air Commodity Interdiction to Joint Force Operations (U), Proceedings of 29th MORS, USAFA, June 1972, Unclassified.



DIMENSIONALITY OF  
FIRE SUPPORT MIX ANALYSIS

calculation consists of establishing the values in an array of enemy and friendly operational functions of interest for each distinguishable homogeneous operational phase of combat, usually referenced to whichever side is exercising the tactical initiative.

For any given situation of friendly/enemy forces, missions, and operational combat phase we are concerned with a weighted sum of terms, one from each enemy operational function, that measures the benefit to each of the impacted friendly functions, or capabilities. This, then, represents a theory of the problem of fire support system mix effectiveness analysis (FSSMEA) that is appropriate to both the direct support role and to the general support operations role, including air delivered fires. The choice of measures of effectiveness differs between the two roles, however. In direct support we need to assess the impact on enemy operational functions from physical damage and suppression that is explicitly referenced to the friendly echelons that are to serve as accounts for benefits accumulation. A convenient system is to measure the increased friendly functional capabilities resulting from the effects of fire in the (friendly)x(enemy) function array (fire, maneuver, C<sup>3</sup>, resupply, medical) introduced, above. This in turn can be related to expected mission success, advance, casualties, security, or other military utility in the situation of mission and phase of operations in which the supported unit finds itself.

Taking the matter of adducing fire support benefits this far, to the military utilities, is probably sufficient for most decision making. The most useful result of an analysis of a ground fire support issue is one that expresses the consequences of the agreed facts and assumptions in the same system of utilities a senior military decision-maker would use if he were actually commanding in the situation visualized. For it is precisely at that point that the analysis permits him to bring all of his own critical faculties to bear in considering the problem. This takes the matter beyond the single point "answer" or "solution" usually produced in a study, which at best has the significance of a revealing example. In this way the analysis can become the senior military manager's own, just as in operations the estimate of the situation and the decision are the commander's own, taking inputs and choices from his various staff.

In the general support operations realm of fire support, alteration in overall mission capability for the supported and opposing echelons is a useful measure, where delayed effects having different time dependencies are involved. Delayed effects are not necessarily of negligible magnitude compared to the prompt exploitable utility of a comparable effort in the DS mode. They are frequently neglected in analyses of their value considered as support fires, however, owing to the complications of reckoning benefits over an extended span of time, geography, and force supported.

In either fire support role, direct support or general support, a key point remains that targets for engagement are distilled, not from enemy

"deployments" or from "combat information", but from planning for operations and for supporting fires, in light of the foregoing. The valuation and costing of fire support and fire support systems must be consistently related to the plans and operations of the friendly forces considered.

It should also be noted that fire support stands out as an area relatively poorly served by analysis because it is truly imbedded in the formulation of operations in combined arms context and is hence truly difficult to express in analytical terms.

Fortified by these spiritual exercises the mini-workshop took up the matter of finding a logical structure of fire support, with the aim of providing a sound scientific basis for future fire support studies that can justify the innovation in materiel and forces, in an analysis driven decision environment, that can bring forth successful operations by smaller forces by improving qualitative change in the Art of War. One can recall in this connection that product improvements are decisive for the larger of two opponents, not for the smaller.

The present FSSMEA capability and activity was viewed as defective in three critical areas.

1. Suppression is handled inadequately or neglected altogether, even in studies of direct support fires.
2. There is no present analytical or experiential basis for formalizing decision algorithms for study of future weapons and forces in which innovation will be needed, for operation of the cheap, rapid computer simulation runs that are essential to acceptability of study findings as regards generality of scenarios, system alternatives, and sensitivity analyses.
3. There is no work underway aimed at overcoming the shortcomings in 1 and 2, above.

With respect to the problem of suppression effects, two significant thoughts emerged - it is a physical and not a mystical body of phenomena that needs to be modeled, fit to available data, remodeled, refit, etc., in a manner exactly similar to the reduction of any body of experience to systematic description, analysis, and manipulation - and that for insufficient reason, the analysis and modeling community has heretofore exempted it from intensive examination.

By way of exception, the work of Lind at Rand Corp. on an extensive body of Marine Corps furnished data from Viet-Nam operations has been reduced to analytical description that, when inserted into a combat model, successfully "predicts" the outcome of a large sample of low level engagements. This work will shortly be published as the AGATE Model. Beyond this, new techniques of modelling on computers described in limited

detail by Horrigan of Horrigan Associates, Chicago, which attempt to develop a general dis-aggregated structure for definition, generation and resolution of very low level (1 man resolution) combat functional events, again describable, calculable, physical phenomena of fire, were asserted to be readily at hand for first fitting the data from the present forms of low level combat, and then generalizing into the future innovations and novel combat environments and forms, and weapons techniques of interest.

As to the needed decision algorithms "indicative" if not "typical" of what commanders and their staffs will make of the situations in which they find their commands, and the capabilities at hand for doing something about it, this too was viewed as a difficult, but essential, and feasible, and hence, indicated area for extensive work with advanced man/machine systems - typified by RAC's ADVICE II "model" of a few years back, which had a number of desirable characteristics. It ran real-time with high quality combat assessments from the Division Battle Model (DBM) which could be driven by new COMANEX parameter values reflective of the combat environment, situation, and capabilities from which we wished to evolve tactical concept, doctrine, and patterns of decision. It used a small team (5) to run the model and the project that used it.

This would give the necessary input to an algorithm generating process for imbedding appropriate military decisions at the combined arms echelons of force, division and above, into an otherwise suitable computer simulation, provided that the virtual decision algorithm folded into the low level simulation used to produce the inputs to the COMANEX post-processor was judged to be acceptable. In the event it was not, new low level simulation runs would need to be made with a problem of comparable difficulty in using appropriate new patterns of decision and combat interactions at the level of micro-tactics. This has always been a problem for the low level combat simulations, but could be handled by simply cutting them off when a traditional militarily reasonable tactical development of events was seen to be departed from. When novel situations of forces and capabilities are posited sufficient to deny use of experientially based judgement and review, the question of keeping the simulation on the rails becomes one of finding out where the track should go and with what gauge the rails should be laid, using methods external to the simulation itself. The approach of using a detailed manual walk-through of the combat action is both obvious and feasible - but must be recognized, accepted, and implemented.

The mini-workshop felt that not all Marine Corps management decisions in fire support require analysis at all levels up to level six in Pugh's classification, and that the lack of analytical technique is most serious at levels 3, 4, and 5. Most specifically, it was accepted that there is little of value to be done to improve the special methods of level six until we are able to tax them with improved quality analyses and inputs from below.

### Recommendations

Three track man/machine effort aimed at combined arms appraisal of fire support.

1. Activate RAC ADVICE II System with DBM (Obtain hardware),  
Shake-down team, design and start experimental program.

1-2 yrs at 6 TMY Level

Review Vector DIVOPS and RAND AGATE Models (others)

To replace DBM in yrs 3,4

1-2 yrs at 1 TMY Level

2. Model and program algorithmic model of physical suppression

Effects of fire within the rifle company.

2-3 yrs at 1-2 TMY Level

3. Then develop new combat model (BONDER 7?), possibly improved "ADVICE III" man/machine interface as basis of faster, cheaper, continued experimental programs.

"Mathematize" decision algorithms with surrogate values obtained across spectrum of appropriate methodological approaches to FSM problems.

### Remarks

User - Don't do Level 3-6 analysis on our Level 2 problems

Us - For the type of work recommended continuity over time and objectives for small good teams gives results. Short, intense disjoint efforts do not.

APPENDIX 12  
SUMMARY OF INVITED PAPERS  
by Gerald J. Lieberman

SUMMARY OF INVITED PAPERS OF THE  
FIRE SUPPORT METHODOLOGY WORKSHOP

Gerald J. Lieberman  
Stanford University

At the very beginning of the workshop it was pointed out that there are three groups of invited attendees; the (military) consumer who submits the problems and is the study recipient; the "fire support" analyst who attempts to provide these solutions using the existing state of the art; and finally the operations research methodology experts who push the frontiers. Each group received an education. The workshop program provided for a description of the fire support system followed by a summary of the existing models and techniques. We were given proposed characteristics of an ideal simulation model which, hopefully, could be developed in the near time framework. A concept of an overall fire support analysis methodology was presented. We were given some promising techniques that are just now being applied to fire support problems. Finally, we heard of some new methodology that may have relevance to fire support.

Harrison and Zimmerman provided some history and background for fire support analysis. The Marine Corps must make decisions pertaining to the following basic problems. What are the USMC fire support requirements? What is the breakdown of fire support requirements for amphibious operations among the three basic systems: air, ground, and waterborne. Within a basic system, what number and type of weapons are necessary to "optimally" meet requirements, year by year into the future. They concluded that the methodology developed to evaluate fire support systems has to



handle not only the "old problems" e.g., target detection, but also has to be flexible enough to handle new tactical concepts, e.g., a mobile armored enemy.

This introduction set the stage for a summary of existing models and techniques in fire support. Dennis divided the fire support system into three functional subsystems: Target Generation, Target Designation, and Target Engagement. The Target Generation Subsystem is composed of the sensors and other processes used to detect the existence of a potential target. The Target Designation Subsystem is the process which acts on the flow of potential targets from the Target Generation Subsystem and decides which should be engaged by supporting arms. The Target Engagement Subsystem involves the delivery of ordnance to targets and the assessment of the damage inflicted. Dennis suggested that there are six levels of analysis of fire support systems which provide a quantitative basis for procurement decisions: 1) Engineering Performance Characteristic, 2) System-Subsystem Performance, 3) Combat Effectiveness, 4) Sensitivity Analysis, 5) Force-Mix Analysis (snapshot), and 6) Time-Phase Force-Mix Analysis. He asserts that almost all of the existing studies are performed at the 1, 2, and 5 levels. He discussed existing fire support system models and classified them as being fire support performance models (non-interacting combat systems) or force as a whole models (too aggregate). In particular he described MAF as an example of the former and Balfram and Vector 1 as examples of the latter. Dennis concluded that studies belonging to both categories are inadequate in their present form.

Generally, the criticisms leveled at existing models by Dennis is valid. However, such models as Balfram and Vector 1 were not developed for fire support purposes, and it is quite possible that they can be suitably modified for this purpose to meet the objections raised.

Schumacher raised some questions about the tentative development plans for fire support by the Marine Corps. How is fire support integrated into the entire mission? How can the Marine Corps put all studies into the same framework? Zimmerman indicated that the Marines must select a fire support model from the existing models or design new ones. Ultimately such a recommendation must appear. He divided methodology development into short term and long term efforts. Under short term efforts is the derivation of an "adequate" target list and a thorough study of suppression. Under long term programs is the exploration of promising techniques, and a study of scenarios and concepts as well as the structure of combat. He then introduced the features that an ideal (impossible) fire support system model should process, quickly passing to the characteristics of an ideal (realistic) model, which he feels can be made available for use with today's state of the art. Such a model would be a pure computer simulation at the marine amphibious force level. It should be resolved at least to the company, battery, sortie levels, which would require the representation of a few hundred units on each side. It should run in about one hour, which should be sufficient for multiple runs. This would permit a sensitivity analysis calling for perhaps 100 variations which could be run on the order of a few weeks. In order to meet the resolution requirements it should make calculations at least once per hour

of combat time and be able to simulate up to 30 days of combat. Terrain should be resolved to a few hundred meters in intense combat areas, which might call for 5,000 area subdivisions. It should be produced within 2-3 years using between 6-9 man years. It should be capable of use after a 3-4 month study. Zimmerman claimed that such a model can be developed today. This is not completely evident, nor is there consensus that such an attempt should now be made.

For example, Bobick and Peters in their paper "Framework for Effective Fire Support Analysis" called for a moratorium on additional model building until a satisfactory fire support analysis structure is developed. They argue that work is required in the area of defining a hierarchy of fire support decisionmakers and the associated measures of effectiveness. They also concluded that no single fire support model can be developed that will fulfill all requirements. Instead they suggested the development of a package of pertinent models. They also called for a data reservoir which provides the necessary information for developing inputs to fire support analysis and for pooling the results of past and future analysis. These conclusions resulted from their proposed framework for fire support analysis in which the principle structure components are the fire support decision interface, the quantitative fire support analysis, and the fire support data reservoir.

A series of papers presenting some promising theories and techniques were given. New spoke of the need for dynamic planning. He presented a few guidelines for "optimizing" the evolution of force mix over time rather than optimizing at some instant in time. This would enable the

phasing in and phasing out of weapons over time, taking into account changes in requirements. New was arguing for a level 6 analysis. In essence, he was proposing treating the fire support problem as an n stage dynamic program, although he never referred to it in these terms. In principle such an approach is desirable, but there is concern about the computational difficulty associated with such an approach.

Pugh introduced the idea of a value driven simulation (or alternatively, an information driven simulation). In the traditional treatment of automated decisions in combat simulations, the decision process is frequently replaced by standardized rules of thumb so simulation can proceed. This is deficient in that actions may be induced by a poor choice of "standardized" rules. Pugh proposed using intuitive decision processes where one evaluates intermediate outcomes using surrogate (or judgmental) values. He likened combat to a game of chess. In both, there is a single objective (to win the battle or game). Yet in chess, players typically assign "surrogate values" to various pieces and at least in early stages may play so as to maximize the difference in the valuations between their pieces and those of their opponents. Such a tactic is suggested for use in simulating combat. Pugh argued that a value driven simulator approach models rational decision process, thereby avoiding dependence on arbitrary decision rules. Although, this idea is certainly worth further exploration, it does have some disadvantages associated with it. All parties to the decision making process have to agree to the "surrogate values". This is similar to the problem in Bayesian analysis of finding suitable (agreed upon) prior distributions and loss functions. Furthermore

since this technique involves "optimizing within modules", there is no guarantee that the final result is "optimal".

Horrigan proposed the use of algorithmic models. He intends to simulate modules that are truly representative of the real world via computer programs that are computationally efficient. These models would operate at the second, third and fourth levels of analysis. It is difficult to assess his technique since he did not present any methodological details, other than some examples where they may be used, e.g., suppression.

Cherry spoke on the role and use of differential models. He described the work being done at Vector Corporation, particularly in the context of Vector 0, Vector 1, DIVOPS, and Vector 2. He indicated that Vector 0, 1, and 2 models were designed for theater type operations, and hence have limited use in fire support analysis. However, he asserted that DIVOPS represented a cross section of Vector 1, which is useful at the division level and has applicability to fire support analysis. Further, Vector 2 will be a high resolution model and may also be useful in fire support. None of the Vector models contain optimization procedures for weapon mix and/or tactics.

The remaining papers presented ideas that were at the forefront of operations research methodology, and hopefully, would be relevant to fire support analysis. Dantzig presented an application of mathematical programming to a problem in ordnance planning. The model represented a large-scale planning and allocation of ordnance and aircraft. The problem was to determine (1) alternative stockpiles of munitions and (2) most effective weapon modules for development, (3) allocation of resources

between aircraft and weapons. Four time periods (10, 30, 50, 100 days) were utilized with decisions made at each time period. The variables treated were sortie types, aircraft, ordnance, targets, delivery conditions, weather, and time period, resulting in over a million possible combinations. The model developed was a convex nonlinear program (piecewise linear function) whose objective function essentially maximized the targets destroyed subject to constraints (35 types) on effectiveness, sortie rates of attrition, budgets, etc. The software characteristics developed could handle 400 equations with over 500,000 variables. Only 1/5th the size is currently needed. This model was time phased with successive optimization. It can be categorized as belonging to the level 2 category of analysis. This model is certainly large scale and is representative of a large class of problems that can be handled similarly. Although such a development optimizes within modules (or stages, or periods) Dantzig argues that such myopic rules may be desirable. In the first place, they can be put into competition with other simulation schemes, and the "better" one chosen. This is the "beauty" of simulation. If such stage by stage procedures are not good, he suggests the alternate scheme of simplifying the model until it can be solved as a dynamic program. This leads to constraints at each stage. The "realistic" problem may then be solved (approximately) by optimizing at each stage subject to these constraints.

Shubik spoke on the "Uses of Game Theory, Gaming, and Model Building in the Study of Fire Support Problems". He stressed the assets and pitfalls relative to the potential use of these techniques in the fire support context. In particular, game theory has been successful in the

solution of oversimplified models, not representative of real fire support problems. Nevertheless, he observed that these oversimplified games are often a good way for starting the model. It often clarifies which variables one should include in the final model. Shubik described the many purposes of gaming, and presented several advantages of man-machine gaming: sponsors and users coming together, introduces the behavioral model problem, and opens the black box to scrutiny. On the subject of modeling he cautioned the analyst to concern himself with data validation, relevance vs realism, sensitivity analysis, aggregation, symmetry, and built in bias. Shubik remarked that it is important to develop analytical models, games, and large scale simulations in parallel rather than in series.

Sobel's paper was concerned with considering an alternate to simulation as a means for solving the fire support problem. In particular, he suggested that dynamic programming be used. He argues that a "head on" solution may result in a level of dimension which is not too dissimilar from that required for the solution of large scale mathematical programming problems. He also indicated that the level of effort may be comparable to that used in a simulation model. He further noted that the fire support problems may be characterized as a network decision model, and network optimization codes of mathematical programs handle large scale problems, and hence may be useful for this purpose. He concluded by presenting some R & D needs in this area: optimization packages for large dynamic decision models, further synthesis of simulation and network optimization, computational solutions of stochastic games, and decision network representation of the fire support system. Whether or not Sobel's proposals are feasible he

does raise a very important issue. We are all quick to use simulation without adequate inspection as to whether or not it is necessary. A distinguished mathematician once remarked that "simulation dulls minds". Sobel is asking us to look before we continue to leap.

The remaining two papers were concerned with simulation. Fishman spoke on simulation as a methodology for fire support systems. He presented and discussed six features of simulation as a tool for studying complex systems, i.e., 1) compression of time, 2) expansion of time, 3) model detail, 4) selection of outputs, 5) control of measurement errors, and 6) control of variation. He discussed this last feature (control of variation) in detail. Generally, this feature refers to the ability to control the pattern of variability in a simulation with resulting decreases in the computer time required to obtain results of a given statistical accuracy. Several variance reduction techniques were presented together with an example to illustrate their use. Such variance reduction techniques may be very important in fire support simulations in that it may enable the user to reduce the amount of machine time required.

Iglehart presented a very new development, a major breakthrough, in simulation. He introduced the regeneration method. The basis of this method is the collection of data during each of a number of regenerative cycles that will be independent and identically distributed. This requires the existence of regeneration points, which do exist in a wide variety of problems. He described methods for "efficiently" estimating the desired parameters of the simulation with prescribed "accuracy". He also gave two approximation techniques for dealing with non-regenerative systems or regenerative systems for which it is difficult to identify the regeneration



points. A major advantage of this technique is the elimination of the need to discover when the system leaves the transient state and enters the steady state. The regeneration method has had important applicability to intermediate size problems (e.g., computer scheduling). Whether or not it will have an impact on fire support simulation models remains to be seen.

The fire support workshop provided a forum for talk, and hopefully will lead to some action. As a methodology participant, I was anxious to hear of new ideas that may have immediate application to fire support analysis. I think that I received a few messages. Large scale systems are present, and so is some useful software. Perhaps then, simulation is not the only technique available for fire support analysis, and we should seriously look at such tools as dynamic programming, game theory, and mathematical programming as alternatives. There do exist some new techniques such as in simulation, which may be useful in fire support analysis. Finally, perhaps we need not always look for overall optimal procedures. Heuristic procedures which sub-optimize at the module level, or which are optimal for simplified versions of the system, may be adequate.

Another main issue that is relevant to the workshop is a recommendation as to how the Marine Corps should proceed in the area of fire support analysis. It would be presumptuous of me to come forth with such a recommendation. However, it appears to me that one model will not suffice. We spoke of different levels of analysis, and it seems reasonable that a model be developed for each level problem, in a hierarchical fashion, with the outputs of one level possibly becoming the inputs to the next. Thus,

a level 2 problem could be run for decision making purposes at this level using one measure of effectiveness, or it can be run for input at level three using a different form of level 2 output. The inputs at level one should represent meaningful and validated parameters of the model. The higher the level the more aggregate the model becomes. In fact, realistic simulations can be constructed primarily at the lower levels.

APPENDIX 13  
SUMMARY OF INVITED PAPERS  
by James G. Taylor

SUMMARY OF INVITED PAPERS  
by James G. Taylor

1. Introduction

The purpose of this paper is to review invited papers presented at the ONR Fire-Support Methodology Workshop. The purpose of this workshop was to establish and evaluate the state-of-the-art for methodology to evaluate fire-support systems and to assist in the fire-support system acquisition process. In other words, the ONR Fire-Support Methodology Workshop was held to establish the state-of-the-art for methodology to assist operations research analysts answer questions such as:

- (1) What methodology should be used to evaluate mixes of fire-support systems?
- (2) How does one determine a "good" mix of fire-support systems, giving consideration to cost constraints, organizational constraints, and time-phasing of system acquisition?

Thus, the workshop was held to determine by what means should OR analysts assist USMC decision makers in future fire-support system-acquisition processes.

The purpose of this paper is to provide the author's assessment of to what degree the above workshop objectives were achieved and to provide a commentary on technical issues. We will try to summarize conclusions reached by workshop participants (i.e. indicate those technical points on which a consensus of agreement was reached) and also try to point out unresolved problem areas.

Another objective of the workshop was the exchange of information among attendees. There were primarily three types of attendees at the workshop:

- (1) military staff members (study consumers),
- (2) operations analysis (OA) practitioners (those who do applied research<sup>+</sup> studies),
- (3) operations research (OR) theoreticians<sup>++</sup> (those who develop theory and techniques).

<sup>+</sup>We are using the term "applied research" in the sense of R. Ackoff (see pp. 7-9 of reference 1).

<sup>++</sup>There was, remarkably, no overlap (except for the author) in these researchers with those who participated in the recent Symposium in the State-of-the-Art of Mathematics in Combat Models (June 1973).

One is struck by the fact that the age of specialization is certainly here.

The conference was centered around research conducted by the Potomac General Research Group (PGRG) for the United States Marine Corps (USMC). The workshop represents Phase II of this research program, with Phase I being for PGRG to assess the state-of-the-art:

- (a) Task 1 - provide a verbal and mathematical description of fire-support systems,
- (b) Task 2 - identify feasible theories and techniques,
- (c) Task 3 - identify current fire-support models.

Reports of Phase I research (see references 6, 8, 31, and 32) were available to some attendees, and the first day of the workshop was devoted to a review of this PGRG work. Based on the results of this research, PGRG presented a tentative development plan.

The following was the overall structure of the workshop:

- (a) first day, background and research summary by PGRG,
- (b) second day, methodology speakers,
- (c) third day, miniworkshops,
- (d) fourth day, summaries of workshop by miniworkshop chairmen (R. Hinkle, A. Goettig, and E. Girard) and J. Lieberman and J. Taylor.

## 2. The USMC Fire-Support Problem

The problem facing the USMC may be stated as

"What should be the mix of fire-support weapon systems year by year into the future?"

New technologies arise and concepts of employment continually change to create shifting fire-support requirements. R. Zimmerman of PGRG pointed out that the above USMC problem generates for the OR analyst the problem of determining what methodology is needed to evaluate fire-support systems and to solve the mix problem.

R. Zimmerman also pointed out that the USMC was faced with a number of pending decisions regarding fire-support systems:

#### Land Fire-Support Systems

1. extended range 155 mm howitzer (maximum effective range extended from 16 km to 24 km),
2. lightweight 8" howitzer (can be airlifted),
3. ammunition requirements,
4. cannon-launched guided projectiles (CLGP) (reflecting USMC concern with Soviet armor threat),

#### Air Fire-Support Systems

1. AVX, V/STOL,
2. naval air-combat fighter,
3. advanced attack helicopter,
4. EO and laser-guided munitions,
5. ammunition requirements,

#### Sea Fire-Support Systems

1. 8" lightweight gun for cruisers (however, cruisers are being phased out of USN),
2. ammunition requirements,
3. CLGP.

In view of the above pending decisions, one clearly sees the USMC requirement for quantitative inputs into the system-acquisition process. In particular, one sees that the USMC must have adequate methodology for evaluating fire-support systems.

### 3. The Fire-Support System Evaluation Process

Since analysis needs generate methodology requirements, it seems appropriate to say a few words about the nature of the fire-support system evaluation process. Although never explicitly stated at the workshop, such aspects are implicit in the PGRG reports (see references 6, 8, 31, and 32).

The structure of analysis (see QUADE and BOUCHER (19)) is shown in Figure 1. For specific USMC questions on fire support:

- (1) identify the feasible courses of action,
- (2) estimate the consequences of each course of action,
- (3) determine the preference structure for outcomes.

In such an analysis it appears that the key questions are

1. What are the USMC fire-support questions?
2. What is the system under study?
3. What criteria should be used to evaluate system alternatives?
4. What models should be used to generate system effectiveness information?

The question of what is the system to be evaluated is particularly important, since its answer has a major impact on the system effectiveness model. To answer the question, "What is the 'best' fire-support system mix?", it appears as though the entire Marine Amphibious Force (MAF) should be the system. For example, the "worth" of artillery fire support can only be assessed by asking, "How did it change the outcome of battle?" To answer this question one must consider the entire MAF. This concept is shown diagrammatically in Figure 2. Moreover, one might even have to consider subsequent army operations in order to determine MAF objectives. Thus, we conclude that in order to evaluate the effectiveness of fire-support systems/units one has to consider combat between division-sized land combat units.

#### 4. The Fire-Support Methodology Problem.

We have seen that the system evaluation process generates methodology requirements, and these in turn generate requirements for system and subsystem models. We have established above that system effectiveness must be evaluated at MAF level (most attendees were in agreement on this point). Thus, one is faced with modelling a very large, complex system.

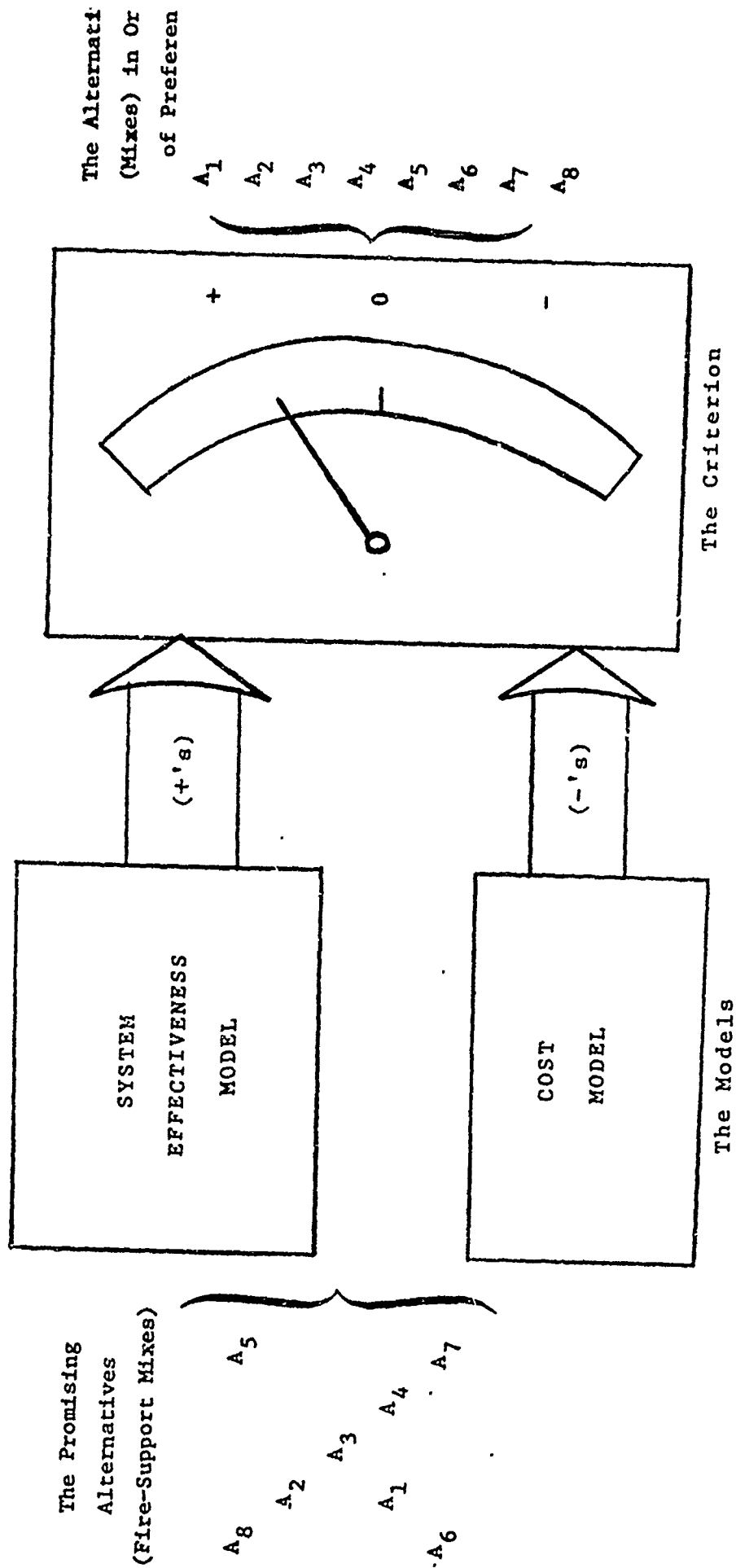


Figure 1. The Structure of Analysis.



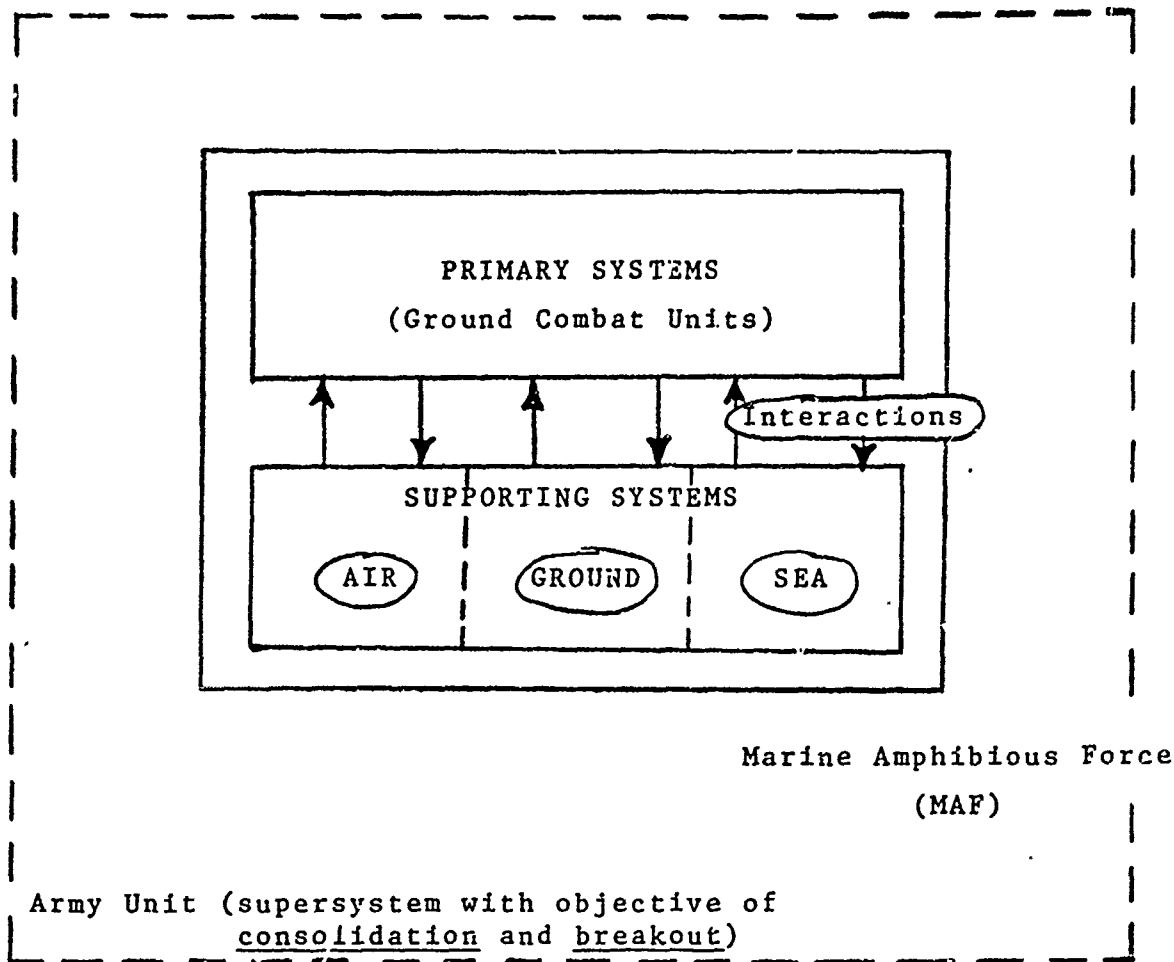


Figure 2. Effectiveness of Fire-Support Systems Evaluated by Considering Operations of Marine Amphibious Force (MAF).

System modelling requirements in turn generate requirements for modelling and optimization theories.

Thus, modelling requirements are generated by the analysis/evaluation process. This idea is depicted schematically in Figure 3. The model requirements themselves depend on

- (1) the system effectiveness information required by the decision environment,
- (2) the system,
- (3) the measures of effectiveness (MOE's).

Hence, the analyst must anticipate what questions will be "typically" asked in the decision environment.

Since one is talking about comparing hardware (which may not physically exist today) alternatives under conditions that don't exist (and cannot be duplicated in any field laboratory, even CDEC<sup>+</sup>), it is clear that models of combat processes and systems must be used. Furthermore, since system effectiveness (SE) must be evaluated at the MAF level, one is talking about the modelling of fairly large combat units (i.e. division-level combat operations). Thus, combat modelling is an important aspect of the methodology problem.

Furthermore, the system acquisition process is a dynamic process which evolves over time. Hence, "dynamic planning" is an integral part of the fire-support problem. This point was emphasized by PGRG (see Appendix A of reference 6 and p. 4-16 of reference 32). The author feels that PGRG has made a valuable contribution in emphasizing this point.<sup>++</sup>

---

<sup>+</sup> United States Army Combat Developments Experimentation Command.

<sup>++</sup> At the 45th National Operations Research Society of America Meeting held in Boston, Mass. in April 1974, the Defense Systems Acquisition Review Committee (DSARC) process was discussed by panelists. A subsequent conversation with one of the panelists, R. Trainor of the Office of the Deputy Chief of Staff for Research, Development and Acquisition, Dept. of the Army, brought out the importance of "dynamic planning". He feels that lack of dynamic planning has been a major shortfall in recent service system acquisition decisions.

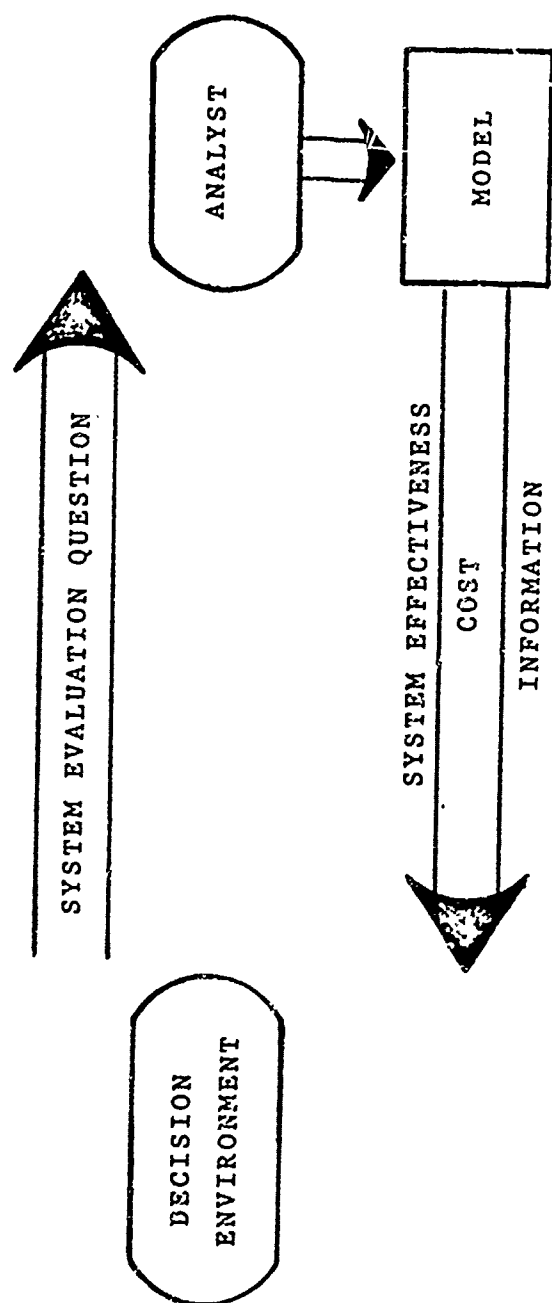


Figure 3. Generation of Modelling Requirements by  
System Analysis/Evaluation Process.

## 5. Summary of First-Day Papers

In summarizing invited papers presented at the workshop we will consider only those aspects which are related to the workshop's objectives. Professor Lieberman has provided a fairly comprehensive summary of all the invited papers to which the reader is directed for a narration of proceedings.

LTC K. P. Harrison provided general background on USMC fire-support interests. This presentation was followed by R. E. Zimmerman, who provided further background material (see Sections 1 and 2 above). Zimmerman enumerated current pending fire-support decisions for the USMC (see Section 2 above). This material (see below) does not appear in the PGRG reports (6, 8, 31, 32). Zimmerman then discussed some old problems of combat modelling related to fire-support evaluation and the new tactical context for USMC fire-support systems.

According to Zimmerman, the old problems of combat modelling were as follows:

- (1) target location errors,
- (2) partial information about the enemy (false targets),
- (3) target detection probabilities (new sensors),
- (4) posture of enemy targets (lethal areas),
- (5) total communications traffic (delays),
- (6) suppressive effects (on fire and maneuver),
- (7) casualties and operational integrity (effectiveness versus unit strength),
- (8) opposed rates of movement (dependence on total force opposed, casualties).

In the opinion of this author Zimmerman gave a good assessment of the contemporary fire-support/combatt modelling problems. He also pointed out the following possible characteristics of the new tactical context:

- (1) mobile armored enemy,
- (2) new anti-tank threat and response,
- (3) massive heliborne assaults,
- (4) defense of airhead,
- (5) powerful SAM and anti-aircraft gunfire,
- (6) loss of major caliber naval gunfire support,
- (7) anti-ship missiles,
- (8) automated command, control, and communications systems,
- (9) extended land campaign,
- (10) tactical nuclear weapons.

Zimmerman suggested that the new tactical context may require new fire-support concepts. Again, the author was quite impressed by Zimmerman's insights into problem areas.

Next, R. V. Dennis of PGRG reviewed selected models and techniques for fire-support analysis (see references 6 and 8; also reference 32).

Dennis found it convenient to consider six levels of analysis:

- 1 - engineering performance characteristics,
- 2 - system/subsystem performance,
- 3 - combat effectiveness (i.e. MAF effectiveness),
- 4.- sensitivity analysis,
- 5 - force-mix analysis (snapshot),
- 6 - time-phased force-mix analysis.

This reviewer was fairly well impressed by this conceptualization of the study process. Dennis then highlighted weaknesses of current methodology for level - 2 analysis: target generation, target designation, and target engagement. Again, the author was impressed by the overall quality of applied military operations research. This presentation seemed quite authoritative.

Dennis then reviewed four "representative" models (see also reference 6). These models were classified into two categories as follows:

fire-support performance models

force-performance models

MAF  
DAFS/CAS

VECTOR-1  
BALFRAM

Dennis (see also reference 6) very appropriately qualified his review by citing the current state of model documentation as reported by SHUBIK and BREWER (21). None of these models have been subjected to a thorough external review. Thus, it is difficult for anybody except an expert to know which claims (if any) of the model developer are true.

The fire-support performance models look at the fire-support system with the exclusion of other systems. Herein lie their major weaknesses: (1) one cannot assess the contribution of the fire-support systems to the outcome of the land battle, and (2) lack of fire-support/maneuver-element interactions. Dennis then reviewed the MAF model in detail and gave a thumbnail sketch of the DAFS/CAS model. He concluded by giving the major limitations of the MAF study: (1) embedding the fire-support system (no fire support/maneuver-element interactions), (2) exclusion of suppression (the dominant effect of fire support), (3) target list not representative, (4) MOE incommensurability, and (5) snapshot approach to system acquisition. The author was quite impressed by this part of the model review. He was less impressed, however, by the review of VECTOR-1 and BALFRAM.

Since there was not enough time for Dennis to go into the details of VECTOR-1 and BALFRAM, the reader should refer to pp. 3-32 to 3-54 of reference 8. Unfortunately, value judgments on the "quality" of these models and/or the methodology that they employ<sup>+</sup> are lacking. In the

---

<sup>+</sup>As noted above, documentation of such models is not complete. Thus, without further information it is difficult to evaluate them. The author, however, has studied supporting documents for both VECTOR-1 (see, for example, references 2, 3, 29, and 30) and BALFRAM (see, for example, reference 14).

opinion of the author, VECTOR-1 represents the state-of-the-art for large-scale (theater-level) combat models. It has incorporated in it numerous new operations research techniques (e.g. Markov renewal process foundation for estimation of Lanchester attrition-rate coefficients for combat between heterogeneous forces), which potentially represent a quantum jump in the state-of-the-art. Such refinements of differential (i.e. Lanchester-type) combat models are significant, since they allow interactions between two heterogeneous forces to be influenced by such factors as target priorities, terrain (i.e. line of sight) features, etc. Unfortunately, such aspects have not been subjected (at this time) to external review. The author feels that these techniques are probably technically sound. I would like to see them reported in the (refereed) open literature, though.

Furthermore, the author does not agree with the criticism of force-performance models given on pp. 4-8 to 4-10 of reference 8 as pertains to VECTOR-1. The basic methodology used to develop VECTOR-1 is certainly adaptable for the study of any fire-support system. We discuss such an adaptation below (i.e. the DIVOPS model). On the other hand, the author believes that BALFRAM is a rather unrefined model compared to VECTOR-1. We do agree with the criticism of force-performance models as pertains to BALFRAM.

To summarize, the author feels that some version of VECTOR-1 (for example, DIVOPS) should be quite seriously considered for fire-support evaluation work. The differential combat model methodology developed by S. Bonder and his colleagues over the years at the University of Michigan and Vector Research, Inc. (see, for example, references 2, 3, 29, and 30) now represents the state-of-the-art in combat modelling (at least in my opinion). This fact, unfortunately, was not brought out at the workshop. The BALFRAM model, however, does not warrant further consideration (although it is probably quite adequate for certain advanced-planning studies).

Next, R. New discussed what he calls "dynamic planning". This methodology essentially consists of developing a dynamical model of the system acquisition process and then applying optimization theory (i.e. (discrete-time) optimal control theory) to this decision-process model. The techniques (i.e. dynamic system modelling, optimal control theory, etc.) are today, of course, quite standard in operations research and engineering. However, the application of such quantitative methodology to applied defense planning problems is new. One point that R. New failed to make in his presentation (see also reference 6) was that he and his colleagues have applied (apparently successfully) such methodology to U. S. Army problems for Mr. Richard Trainor of the Office of the Deputy Chief of Staff for Research, Development and Acquisition, Dept. of the Army. This fact was communicated verbally to the author by R. New. Furthermore, the author has discussed such problems with Trainor and heard him stress their practical importance. Thus, R. New has addressed an important facet (intertemporal planning) of the system acquisition process. It was indeed a shame that New did not explain that the example of "dynamic planning" that he gave was taken from an actual U. S. Army study. There were some questions raised about computational feasibility by workshop attendees. Private discussion between the author and R. New revealed that he has apparently solved several fairly large-scale problems so that his approach appears to be computationally feasible.

G. Pugh then presented what he calls "a value-driven and information-oriented approach to combat simulation." This material is very definitely on the frontier of the optimization of combat dynamics in military operations research to which Pugh has been an active contributor (see, for example,



references 15 through 18). Pugh essentially models time-sequential tactical resource allocation as a rational decision process and considers combatant information structures. Optimization (i.e. time-sequential game) theory is applied to obtain "optimal" time-sequential combat strategies. Because of computational limitations, an aggregated (i.e. low-resolution) model of the combat dynamics is required. In other words, this approach is incompatible with a high-resolution combat operations model.

It has been the author's experience (see, for example, references 24 and 25) that very simple-looking time-sequential combat games are very difficult to completely solve. Pugh (16) has introduced what he calls Lagrange dynamic programming for time-sequential combat games. (Unfortunately, counterexamples have been developed for his algorithm (see GOHEEN (9)).) In the present context if one recalls the well-known marginal value interpretation for Lagrange multipliers (see, for example, TAYLOR (24, 25)), one obtains a motivation for Pugh's so-called surrogate values. Pugh then presented his own motivation for such surrogate values for optimizing "local" combat decisions. His approach appears to be a heuristic approach to "decomposition" with adjoint (or dual) variables being judgmentally determined rather than evolving according to the usual adjoint system. Although Pugh's approach is definitely not equivalent to determining optimal (in the game-theoretic sense) combat strategies, it may provide a very good model of actual human decision making. There is the problem of determining the surrogate values, however. This author feels that Pugh's ideas are very promising and would like to see further results (especially computational studies).

Pugh also considered imperfect state information for combatants in time-sequential combat games. Essentially all such optimization studies reported in the OR literature have considered only perfect state information. Pugh, however, has considered the modelling of information structures (not necessarily the same for opposing combatants) and optimization of combat strategies under such conditions. Pugh is certainly to be commended for his extension of the state-of-the-art for combat analysis. (The reader should note that both DIVOPS and VECTOR-1 give consideration to such information structures. It was indeed remarkable that both Pugh and Bonder have apparently been considering the same conceptual modelling problems.) Pugh's conceptual work has led to the development of a "value-driven and information-oriented" combat simulation called TAC COMMANDER, which appears to be an outgrowth of the TAC CONTENDER model<sup>(28)</sup>. In summary, the author was quite impressed by Pugh's work and feels that in some form it very definitely should be pursued in the future.

T. Horrigan next discussed what he calls "algorithmic models." Unfortunately, he did not give enough information for this writer to even ascertain what his approach is. Unless this approach can be better explained, the author can see little use for it.

Finally, R. Zimmerman presented a tentative development plan. This plan was essentially a "straw-man" proposal for future research. The tentative development plan consisted of both a near-term program and a longer-term (FY76-79) program. The near-term program seemed quite reasonable to this author. The longer-term program was based on specifications given for an idealized operational model. These specifications appeared to be quite reasonable to this reviewer, although he is quite skeptical about PGRG's proposed approach (i.e. value-driven simulation/algorithmic programs). (The idealized operational model described by

by Zimmerman bore a remarkable resemblance to the description of DIVOPS (a VRI model) (see next section.) A substantial number of workshop attendees felt, however, that any tentative development plan was "premature".

To summarize, the first day papers were all (with one exception) quite relevant to the fire-support methodology problem. One of my major disappointments was that I didn't see the state-of-the-art for modelling combat operations of MAF-sized units (see Section 3 above) either addressed or assessed.

#### 6. Summary of Methodology Papers

Unfortunately, not all the methodology papers in this reviewer's opinion were related to the workshop theme. However, both W.P. Cherry's and M. Shubik's presentations were particularly relevant.

First of all, J. Bobick of SRI spoke on concepts for overall fire-support analysis methodology. Although he made several good points, the reviewer failed to detect the same in-depth knowledge of fire-support problem areas as shown by R. Zimmerman and other PGRG analysts.

Next, G. Dantzig spoke on mathematical programming and its role in fire-support analysis. The author does not feel that Professor Dantzig achieved his goal. He spoke on a "new" version of the old "distribution of gunfire" problem, which is over 20 years old (see, for example, references 4, 5, and 13). Although one did obtain some idea of current large-scale mathematical programming computational capabilities for a particular structure of tactical allocation problem, the role of mathematical programming for obtaining insights into optimal time-sequential combat strategies or into optimal system-acquisition policies was not addressed. The author would have preferred Professor Dantzig to have talked about DYGM (see reference 12) and current computational capabilities for (time-sequential) combat games.

The author was quite impressed by W. P. Cherry's presentation on the role of differential (i.e. Lanchester-type) combat models. In my opinion a reader may take such a Vector Research, Inc. (VRI) model as being the state-of-the-art for (large-scale) combat operations modelling. Cherry reviewed the development of such models and highlighted in particular a new division-level combat model called DIVOPS. This model is very comprehensive (for example, the information structures of the combatants are considered), and it apparently contains (at least in my opinion) all relevant factors required for fire-support analysis.

The author feels that the DIVOPS and VECTOR-i models represent possibly a quantum jump in the state-of-the-art for such combat models, primarily through the development of refined attrition-rate coefficient-estimation methodology (both for combat between heterogeneous forces and for indirect-fire weapons) (see TAYLOR and BROWN <sup>(26)</sup> for background and further references). Bonder and associates have apparently developed new powerful attrition-rate coefficient methodology based on the theory of Markov renewal processes and consider such factors as the line of sight process (terrain modelling), target acquisition (including target priorities), target selection, etc. (see references 29 and 30). However, this author has two reservations about the VRI work: (1) the new attrition-rate coefficient methodology (which potentially is a quantum jump in the state-of-the-art), i.e. the Markov renewal process methodology, has not been externally reviewed or been published in the open literature, and (2) significant combat interactions are possibly not preserved by this attrition-rate methodology which apparently considers each side as firing on passive targets. Overall, however, this author was quite impressed by the technical caliber of the VRI work. He recommends that the DIVOPS model be seriously considered for fire-support system evaluation.

M. Shubik next discussed the uses of game theory, gaming, and model building in the study of fire-support problems. Although he did not talk about the specifics of the fire-support evaluation problem, Shubik did address many general problem areas and gave many insights. There were two aspects of human decision-making behavior that Shubik addressed: (1) the decision-making behavior of combatants in the MAF scenario, and (2) the decision-making process in system acquisition. He pointed out that no human factors experts were apparently participating in the workshop and stressed the importance of more work on experimental gaming (see also Appendix E of reference 6). Shubik's remarks are particularly important when one considers the great extent that combat modelling depends on understanding human behavior (both rational and irrational). Finally, Shubik discussed various general principles of modelling. One phrase that I vividly remember is "relevance versus realism". Shubik pointed out that unsophisticated clients invariably confuse relevance with realism. (This certainly was an insightful comment.)

G. Fishman then discussed simulation. Although his presentation addressed seemingly important aspects of simulation, it did not consider current problem areas of combat simulation. I would have much preferred to have heard an expert on combat simulation. For example, no mention was made of approaches to and problem areas of terrain modelling. I seem to recall hearing that many current combat simulations spend a lot of time computing line of sight for target detection. Thus, a major problem area of combat simulation (i.e. terrain representation) was ignored. It would have been more appropriate to have heard about the current state-of-the-art for combat simulation and have models like DYN-TACS, CARMONETTE, ASARS-II, SIAF, etc. discussed. Along this line, it appears that currently Monte Carlo simulation is a viable modelling technique for combat operations of units the size of company and battalion.

## 7. Summary of Miniworkshops

The three miniworkshops were held on Wednesday and were as follows:

- (1) target generation and designation - R. Hinkle (session chairman),
- (2) target engagement - A. Goettig,
- (3) mix analysis - E. Girard.

The author first attended the target engagement session and then the one on target generation and designation. Little needs to be said about the target engagement miniworkshop, since its chairman, A. Goettig, has provided comprehensive documentation of its sessions. It was agreed upon in this miniworkshop that target neutralization/suppression is the primary thing accomplished by fire support and that the modelling of the suppression process is a major deficiency in the existing state-of-the-art. A high priority should be placed upon understanding this phenomenon.

The author also briefly attended the target generation and designation miniworkshop. It was agreed upon that the key process to understand is how an operationally generated target list evolves over time and depends on fire-support-system/maneuver-element interactions. The following conclusions/recommendations were agreed upon:

- (1) threat studies needed,
- (2) any simulation model of fire support should be embedded in combined-arms operations,
- (3) there is a need for a scientific review of the alternating Markov renewal process utilized in VECTOR-1/DIVOPS,
- (4) there is a need for an in-depth study of the threat generation process (including command, control, and communications, intelligence data processing, and inference).

## 8. Overall Summary

In this section we will give an overall summary of the workshop and also will give some comments by the author.

A significant combat interaction that must be preserved in any system model is that between the maneuver element and the fire-support target list.

The ground combat interaction (i.e. combat between ground units) and also the effects of fire support itself have a significant effect on the evolution of the target list. This interaction was not preserved in the MAF study (see pp. 4-5 and 4-6 of reference 8).

More than a single model is required. Model requirements are as follows:

- (1) should be generated by considering the question, "What are typical fire-support questions to be asked in the decision environment?"
- (2) large-scale combat system (MAF) to be modelled, i.e. the level-3 of PGRG,
- (3) produce output relating to MAF mission accomplishment (i.e. information on MOE's),
- (4) capable of allowing sensitivity analysis,
- (5) capable of being interfaced with system acquisition modelling (level-6 analysis).

The major modelling issues are:

- (1) determination of the appropriate level of detail to preserve significant combat interactions while maintaining computational feasibility,
- (2) relevance versus realism,
- (3) target neutralization (i.e. suppressive effects),
- (4) information structure for tactical decision makers.

Unfortunately, the state-of-the-art for modelling combat operations of MAF-sized units was not assessed in the workshop or in PGRG reports.

Three current approaches to the modelling of MAF-sized combat units are as follows:

- (1) high-resolution Monte Carlo simulation (examples: DYN-TACS, CARMONETTE, ASARS-II) (apparently not feasible except for "slice" of the battlefield),
- (2) hierarchy of models à la COMCAP III (see reference 27) (COMANEX provides the interface between CARMONETTE and DBM),
- (3) differential models
  - (a) analytical development of attrition-rate coefficients - DIVOPS,

(b) "empirical" development of attrition-rate coefficients (i.e. use of high-resolution Monte Carlo simulation and coefficient pre-processor to estimate loss rates) - DMEW (inputs pre-processed by PROCS).

It appears that only a few two approaches are really feasible for fire-support effectiveness studies with the specifications given above. Additionally, process modelling problem areas appear to be

- (1) target neutralization (i.e. suppression),
- (2) information structure for tactical decision making
  - (a) perceived system states  $\begin{cases} \text{RED} \\ \text{BLUE} \end{cases}$ ,
  - (b) temporal variations,
- (3) tactical decisions
  - (a) descriptive,
  - (b) prescriptive.

Of the above process modelling problem areas, suppression is the main one, since it is the major aspect of target neutralization by supporting arms. There was overwhelming agreement that suppression should be included in any fire-support system model. However, it is not likely that one would ever obtain other than judgmental data (but based on actual combat experiences) on suppression. This is an area of recent research activity, for example, at the Naval Weapons Center at China Lake, California. Further references are to be found in A. Goettig's miniworkshop report.

To the uninitiated, the computation of hit and kill probabilities is often considered to be a trivial matter. Such is far from the truth (see, for example, ECKLER and BURR<sup>(7)</sup>). One technical problem area that did not receive enough attention at the workshop was damage assessment for supporting-arms (e.g. artillery) fire. In particular, a fairly rough approximation was suggested by PGRG on pp. 3-27 to 3-29 of the Task 1



Phase I report (31). Much better models have been developed (see, for example, SNOW<sup>(22)</sup>), with computer programs being available <sup>+</sup> (see references 10 and 23).

One topic of current interest (actually quite an old problem) is the modelling of intelligence, i.e. perceived system states (see Figure 4 and SHEPHARD (20)). Relatively little has apparently been done (except for the working of G. Pugh) to combine such intelligence modelling with the modelling of tactical decision making. As noted above, tactical decision making may be treated either descriptively or prescriptively. Moreover, tactical decision making may "drive" the entire combat operations model.

Consequently, as emphasized by Shubik more gaming (interpreted broadly) work is needed (see also Figure 5). In this author's opinion, optimization of the tactical decision making process is only computationally feasible for very aggregated deis or for "decomposed" systems.

Finally, let the author make his own suggestions regarding the PGRG tentative development plan. They are as follows:

- (1) consider DIVOPS,
- (2) consider a hierarchy of models à la COMCAP III (see reference 21),
- (3) consider Pugh's value-driven and information-oriented combat simulation approach for only selected time-sequential fire-support allocation decisions (i.e. build a different model from the system effectiveness model for developing such insights),
- (4) model the dynamics of the system-acquisition process before trying to optimize this process (i.e. before considering New and Mylander's "dynamic planning").

With respect to this last suggestion, many participants in the workshop seemed to forget that in order to apply optimization theory one must have a process model with decision variables.

---

<sup>+</sup>Unfortunately, it appears as though the computer programs developed by SNOW and RYAN (23) were partially misapplied in the MAF study.

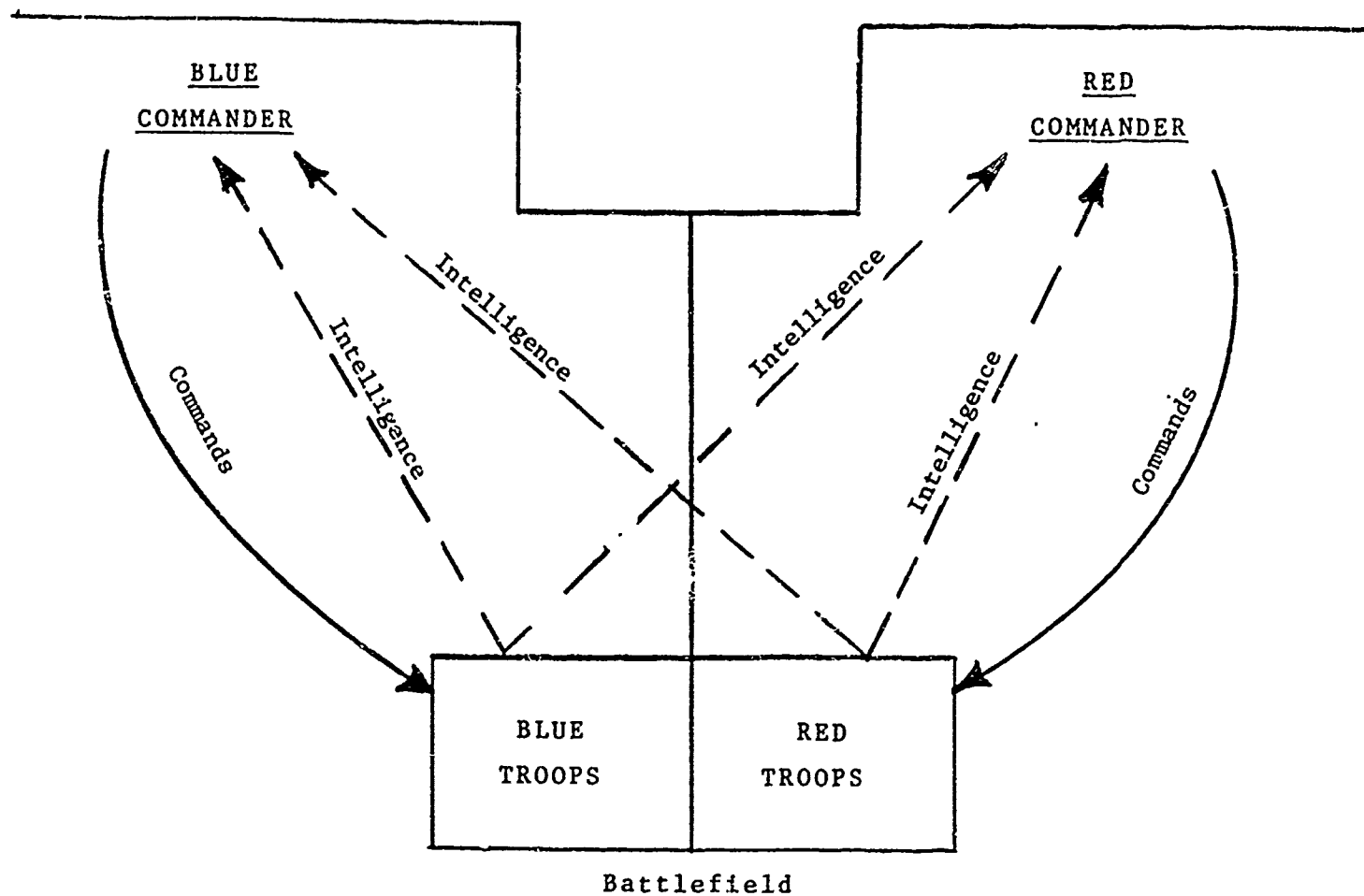


Figure 4. Modelling of Perceived System States  
(After SHEPHARD<sup>[20]</sup>).

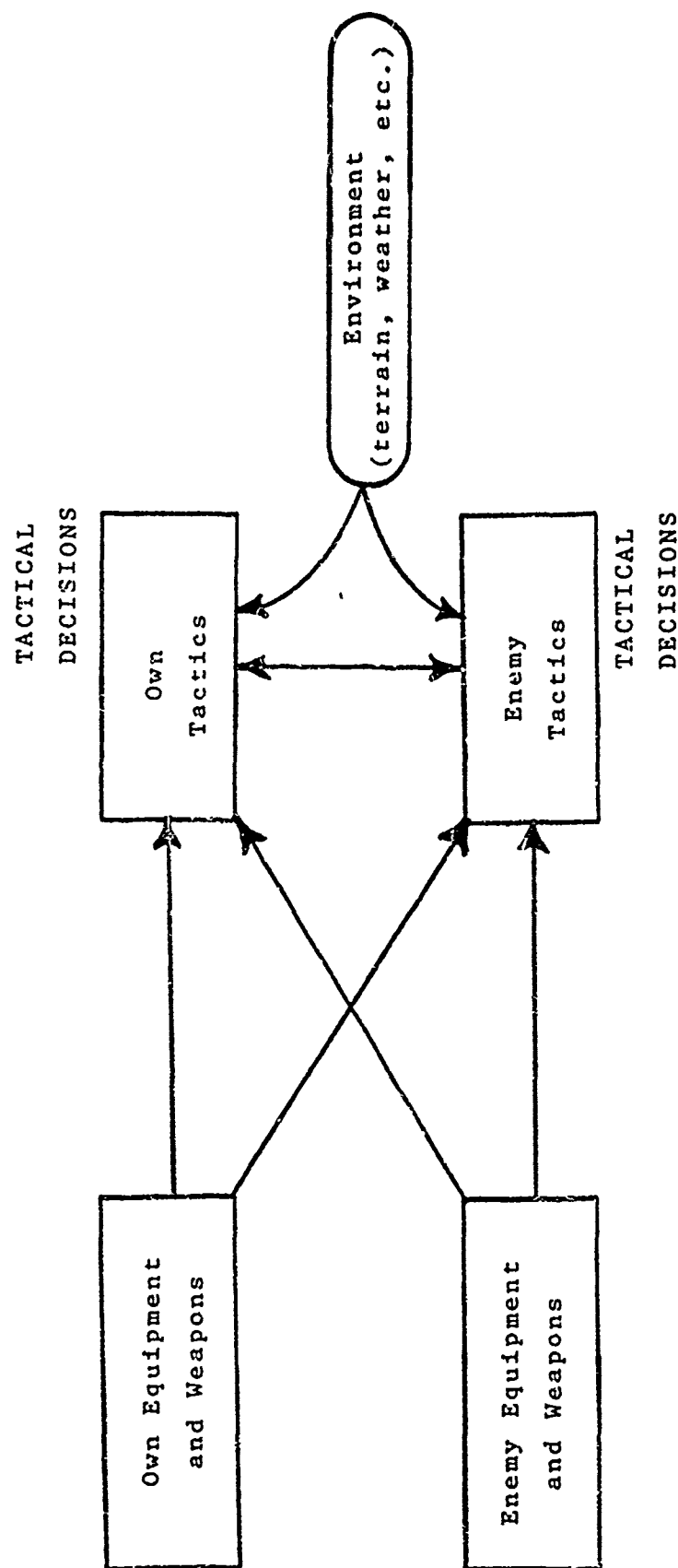


Figure 5. Modelling Tactical Decisions  
(After SHEPHARD [20]).

## 9. Significant Omissions

Based on his participation in the workshop and on his own reflections since then, the author feels that significant omissions of important topics were the following:

- (1) modelling of combat operations (at least some overview was needed),
  - (2) terrain line-of-sight process modelling,
  - (3) intelligence modelling,
- and (4) communications modelling (see, for example, KLEINROCK<sup>(11)</sup>).

## REFERENCES

1. R. Ackoff, Scientific Method: Optimizing Applied Research Decisions, John Wiley, New York, 1962.
2. S. Bonder and R. Farrell (Editors), "Development of Models for Defense Systems Planning," Report No. SRL 2147 TR 70-2 (U), Systems Research Laboratory, The University of Michigan, Ann Arbor, Michigan, September 1970.
3. S. Bonder and J. Honig, "An Analytic Model of Ground Combat: Design and Application," Proceedings U.S. Army Operations Research Symposium 10, 319-394 (1971).
4. J. Bracken and G. McCormick, Selected Applications of Nonlinear Programming, John Wiley, New York, 1968 (see Chapter 2).
5. G. den Broeder, R. Ellison, and L. Emerling. "On Optimum Target Assignments," Opns. Res. 7, 322-326 (1959).
6. R. V. Dennis, "Fire Support Requirements Methodology Study, Task 2 Phase I, Review of Theories and Techniques Applicable to Marine Fire Support," Potomac General Research Group (Contract M00027-74-0042), July 1975.
7. A. R. Eckler and S. A. Burr, Mathematical Models of Target Coverage and Missile Allocation, Military Operations Research Society, 1972.
8. W. W. Edwards, R. V. Dennis, and R. E. Zimmerman, "Fire Support Requirements Methodology Study, Task 3 Phase I, Survey of Programmed and Operational Models for Evaluating the Fire Support System," Potomac General Research Group (contract M00027-74-0042), July 1975.
9. L. Goheen, "Selections of a Method to Solve the n-Stage Game in BALFRAM," Technical Note NWRC-TN-59, Stanford Research Institute, Menlo Park, California, August 1975.
10. K. Harris, R. N. Snow, and J. R. Lind, "FAST-VAL: Target Coverage Model," RM-4567-OR, The RAND Corporation, Santa Monica, California, March 1966.
11. L. Kleinrock, Communication Nets, McGraw-Hill, New York, 1964 (reprinted by Dover Publications, Inc., New York, 1972).
12. Z. Lansdown, G. Dantzig, R. Harvey, and R. McKnight, "Development of an Algorithm to Solve Multi-Stage Games," Control Analysis Corporation, Palo Alto, California, May 1973.
13. A. S. Manne, "A Target Assignment Problem," Opns. Res. 6, 346-351 (1958).
14. E. H. Means et al., "BALFRAM User Manual for the Staff of the Commander in Chief Pacific," Technical Note NWRC-TN-52, Stanford Research Institute, Menlo Park, California, September 1974.
15. G. Pugh, "Lagrange Multipliers and the Optimal Allocation of Defense Resources," Opns. Res. 12, 543-567 (1964).
16. G. Pugh, "Theory of Measures of Effectiveness for General-Purpose Military Forces: Part II. Lagrange Dynamic Programming in Time-Sequential Combat Games," Opns. Res. 21, 880-906 (1973).

17. G. Pugh and J. Mayberry, "Analysis of General-Purpose Force Procurement," Paper 59, Lambda Corporation, August 1971.
18. G. Pugh and J. Mayberry, "Theory of Measures of Effectiveness for General-Purpose Military Forces: Part I. A Zero-Sum Payoff Appropriate for Evaluating Combat Strategies," Opns. Res. 21, 867-885 (1973).
19. E. Quade and W. Boucher, Systems Analysis and Policy Planning, American Elsevier, New York, 1968.
20. R. W. Shephard, "War Gaming as a Technique in the Study of Operational Research Problems," Operational Research Quarterly 14, 119-130 (1963).
21. M. Shubik and G. Brewer, "Models, Simulations, and Games - A Survey," R-1060-ARPA/RC, The RAND Corporation, Santa Monica, California, May 1972.
22. R. Snow, "FAST-VAL: A Theoretical Approach to Some General Target Coverage Problems," RM-4566-PR, The RAND Corporation, Santa Monica, California, March 1966.
23. R. Snow and M. Ryan, "A Simplified Weapons Evaluation Model," RM-5677-1-PR, The RAND Corporation, Santa Monica, California, May 1970.
24. J. Taylor, "Lanchester-Type Models of Warfare and Optimal Control," Naval Res. Log. Quart. 21, 79-106 (1974).
25. J. Taylor, "Target Selection in Lanchester Combat: Heterogeneous Forces and Time-Dependent Attrition-Rate Coefficients," Naval Res. Log. Quart. 21, 683-704 (1974).
26. J. Taylor and G. Brown, "Canonical Methods in the Solution of Variable-Coefficient Lanchester-Type Equations of Modern Warfare," Opns. Res., to appear (Vol. 24, No. 1, 1976).
27. USA Concepts Analysis Agency, "Tabulation of Models of Interest to USACAA," Bethesda, Maryland, June 1975.
28. USAF Assistant Chief of Staff, Studies and Analysis, "Methodology for Use in Measuring the Effectiveness of General Purpose Forces, SABER GRAND (ALPHA)," March 1971.
29. Vector Research, Incorporated, "Analytic Models of Air Cavalry Combat Operations, Vol. 1," Ann Arbor, Michigan, May 1973.
30. Vector Research, Incorporated, "VECTOR-1, A Theater Battle Model," WSEG Report 251, Ann Arbor, Michigan, June 1974.
31. R. E. Zimmerman, R. V. Dennis, and W. W. Edwards, "Fire Support Requirements Methodology Study, Task 1 Phase I, Verbal and Mathematical Description of the Marine Corps Fire Support System," Potomac General Research Group (contract M00027-74-0042), April 1975.
32. R. E. Zimmerman, R. V. Dennis, and W. W. Edwards, "Fire Support Requirements Methodology Study, Phase I Summary Report," Potomac General Research Group (contract M00027-74-0042), July 1975.